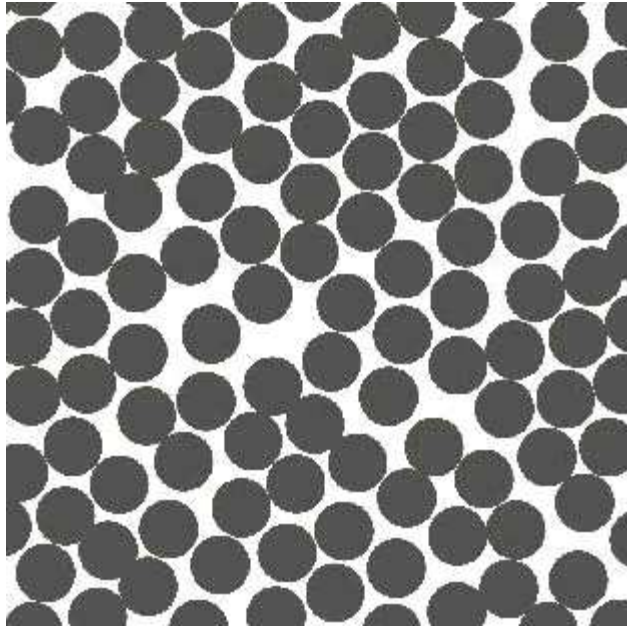


Image Analysis



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Hands On 2009

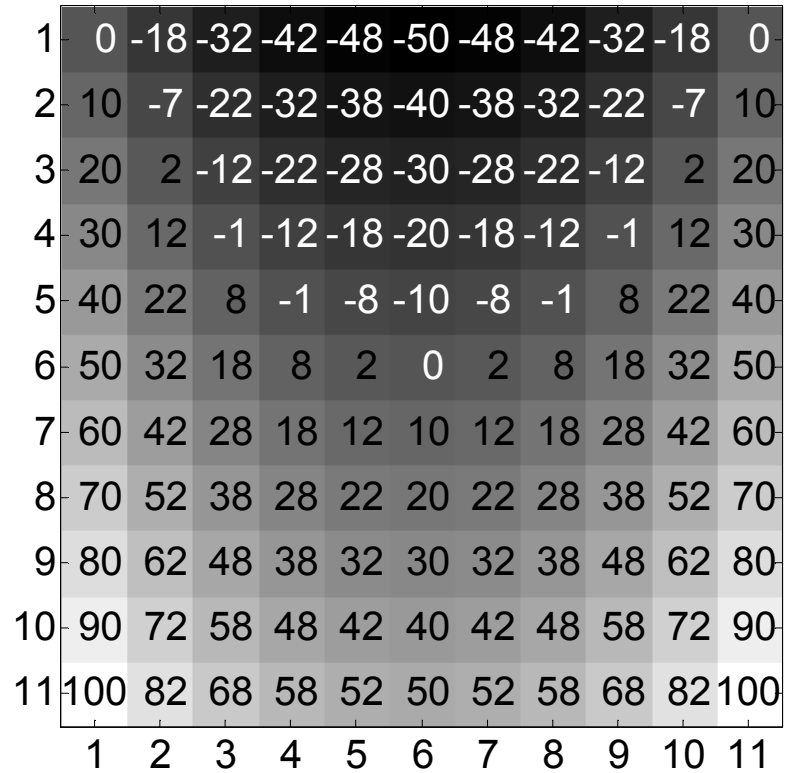
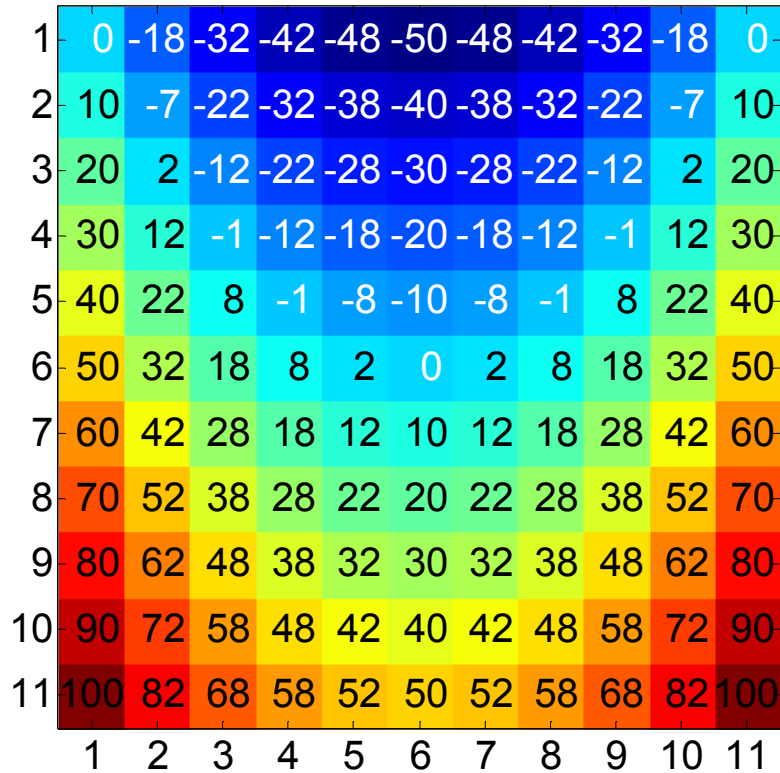
Introduction

- The brain is the best image analyzer.
- If you *cannot* do it with your brain, you *cannot* do with a computer.
- If you *can* do it with your brain, you can rarely do it with a computer.
- Making you image better will pay off.

Outline

- Image Analysis tools
 - Fast Fourier Transform (FFT)
 - Convolution and Cross-correlation
- Particle Imaging Velocimetry (PIV)
- Particle Tracking
- Oscillatory Image Demodulation (OID)

Image



I_{nm}

FFT

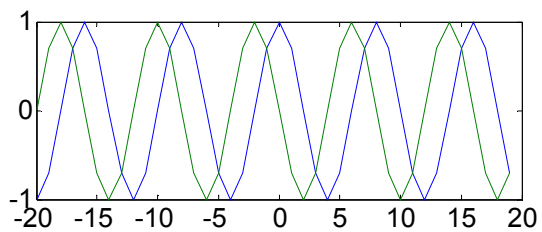
- Fast Fourier Transform

$$F_{nm} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} f_{kl} e^{-2\pi i(kn/K + lm/L)}$$

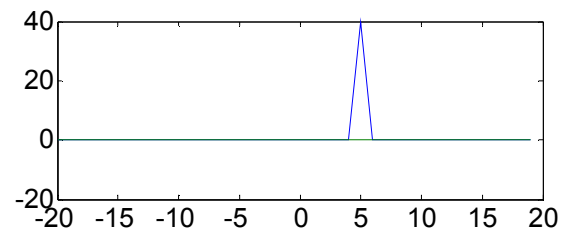
$$f_{kl} = \frac{1}{NM} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} F_{nm} e^{+2\pi i(kn/N + lm/M)}$$

FFT

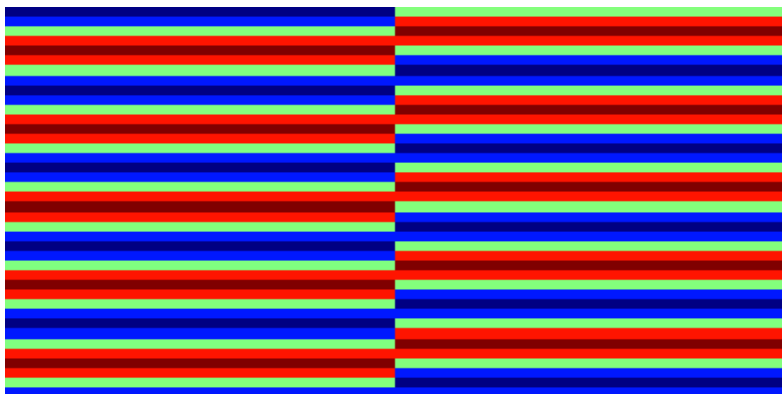
$$f_k = e^{2\pi i k n_0 / N}$$



$$F_n = N\delta_{nn_0}$$



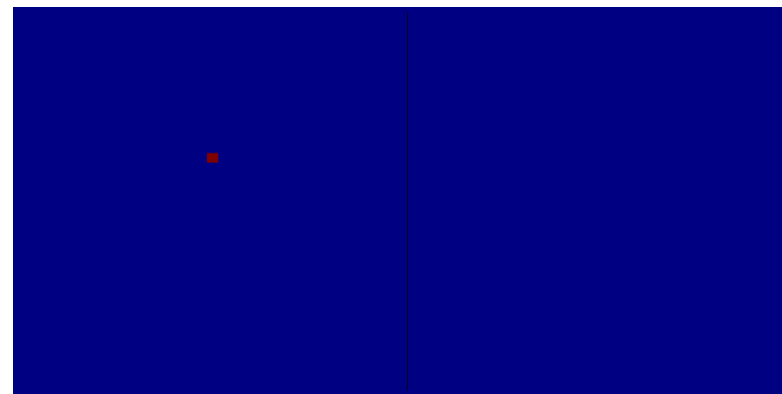
$$f_{kl} = e^{2\pi i k n_0 / N}$$



Re

Im

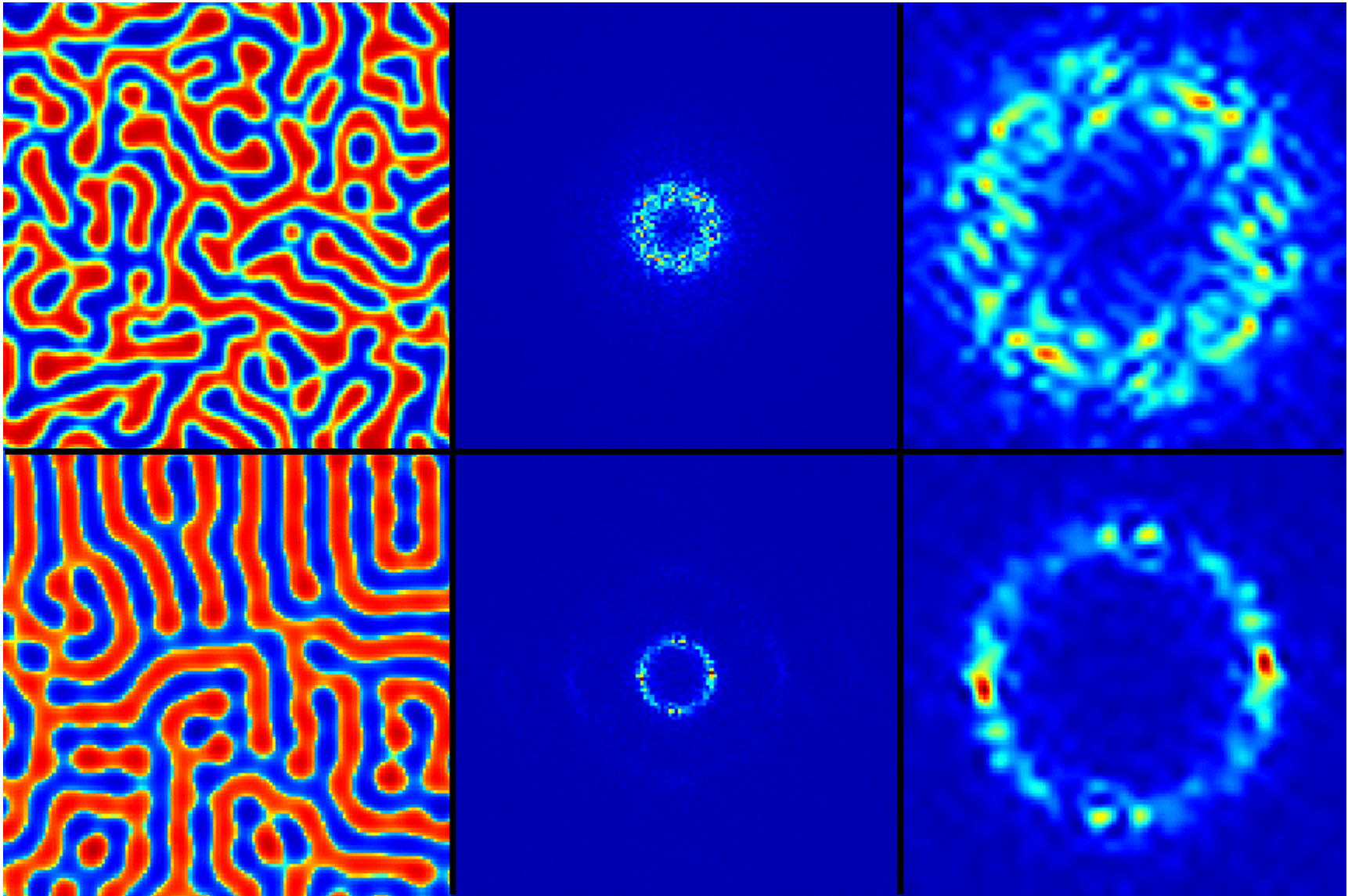
$$F_{nm} = NM\delta_{nn_0}$$



Re

Im

FFT



Convolution

- Convolution and Cross Correlation

$$C(f, g)_{nm} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} f_{kl} g_{(n-k)(m-l)}$$

$$X(f, g)_{nm} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} f_{kl}^* g_{(n+k)(m+l)}$$

Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

6

Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

2 6

Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

7 2 6

Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

7 7 2 6

Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

4 7 7 2 6

Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

3 4 7 7 2 6

Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

1 3 4 7 7 2 6

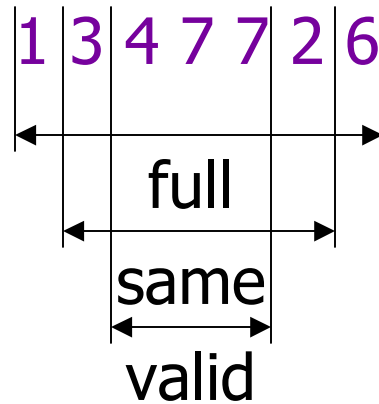
Convolution

1 3 2 1 3

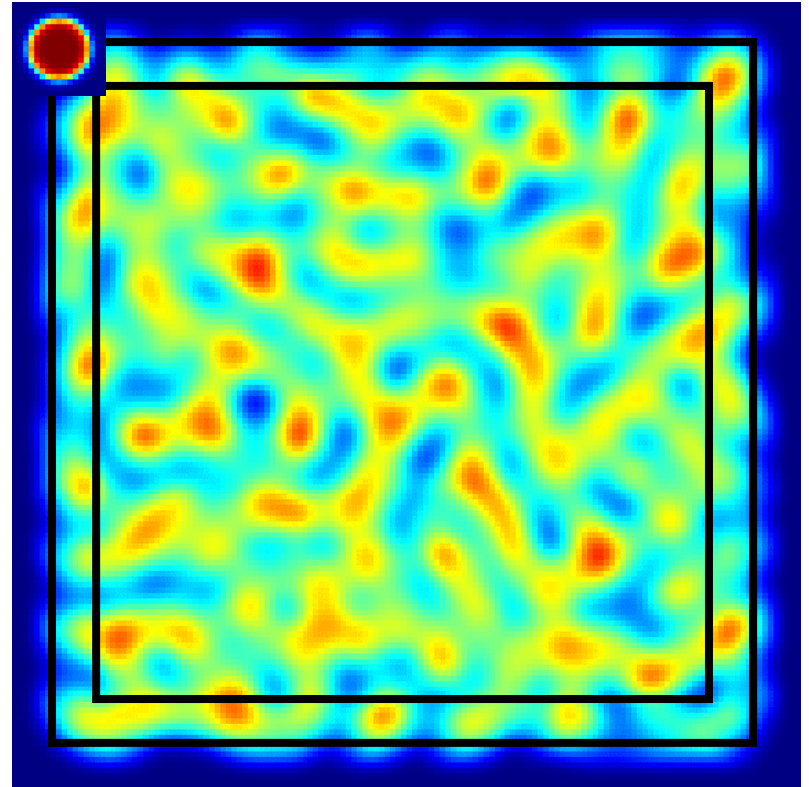
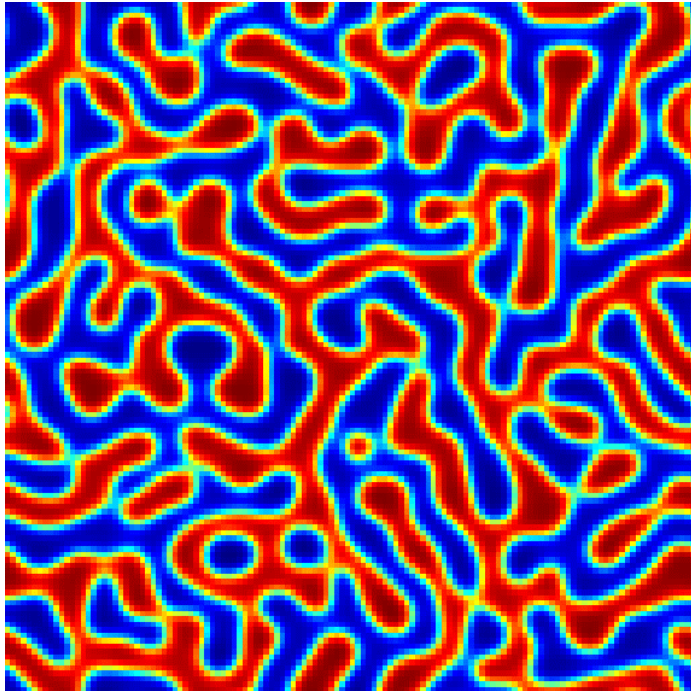
1 0 2

1 3 2 1 3

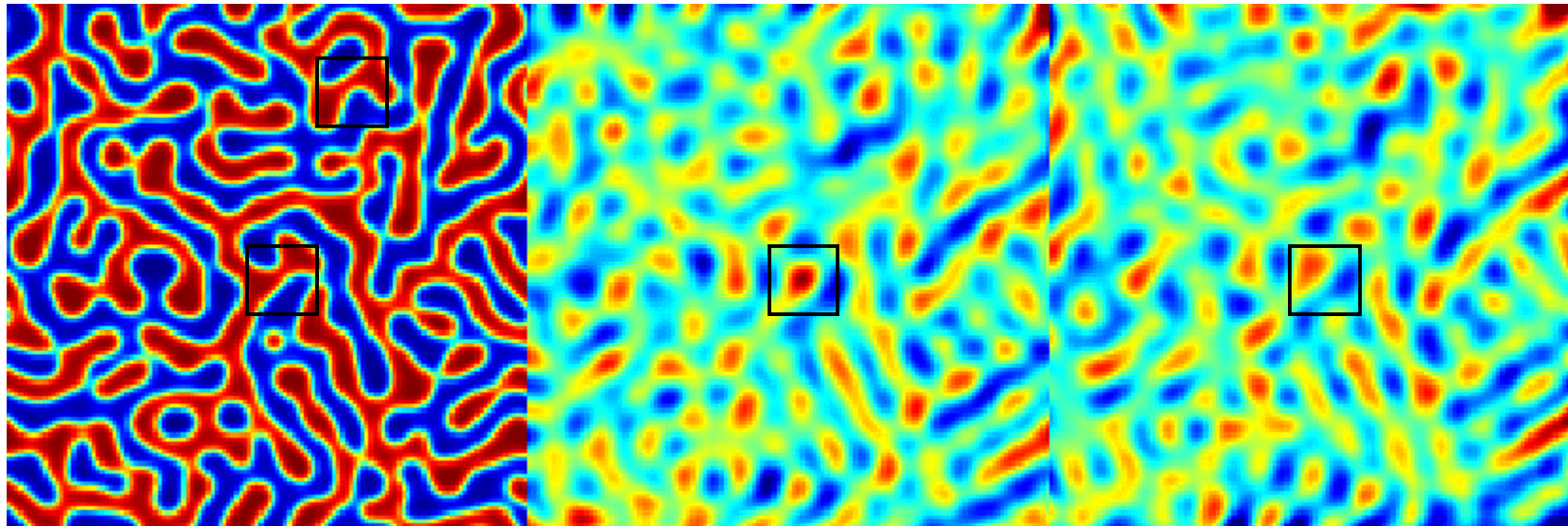
2 0 1



Convolution



Correlation vs. Convolution



Correlation

Convolution

Least-Square Fit

- Minimum Squared Difference

$$\chi^2(I, p)_{nm} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} [I_{kl} - p_{(k+n)(l+m)}]^2$$

$$\chi^2(I, p)_{nm} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} [I_{kl}]^2 - 2I_{kl} p_{(k+n)(l+m)} + [p_{(k+n)(l+m)}]^2$$

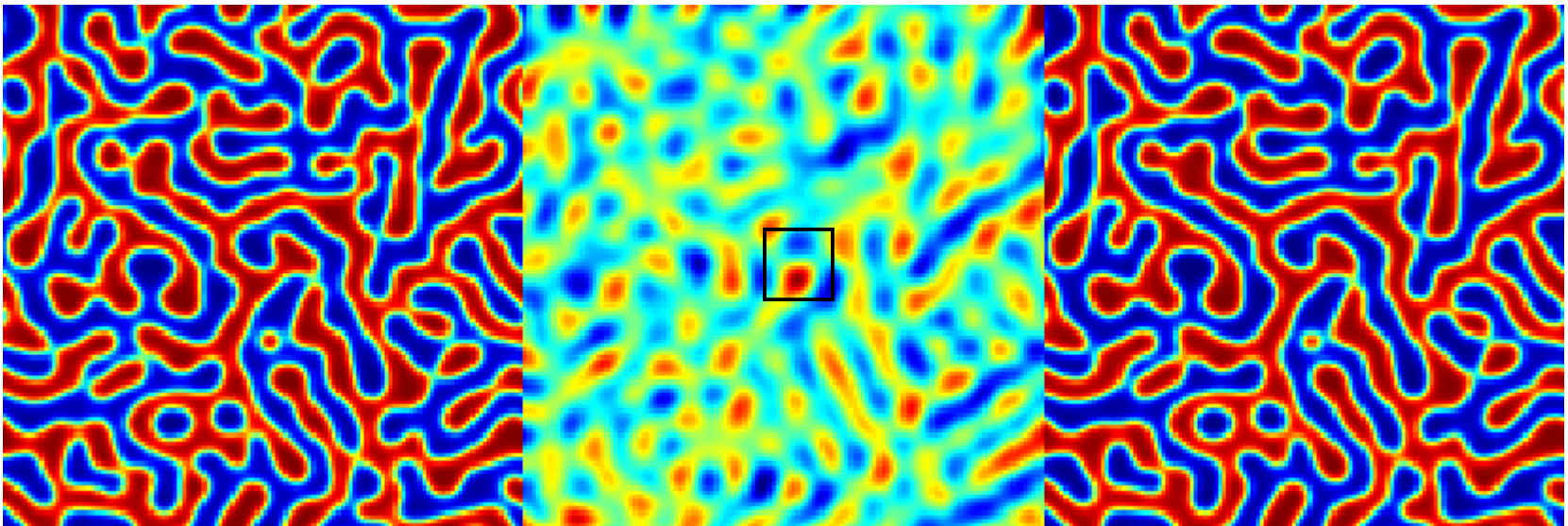
Convolution

$$\min_{nm} \{ \chi^2(I, p)_{nm} \}$$

Particle Imaging Velocimetry

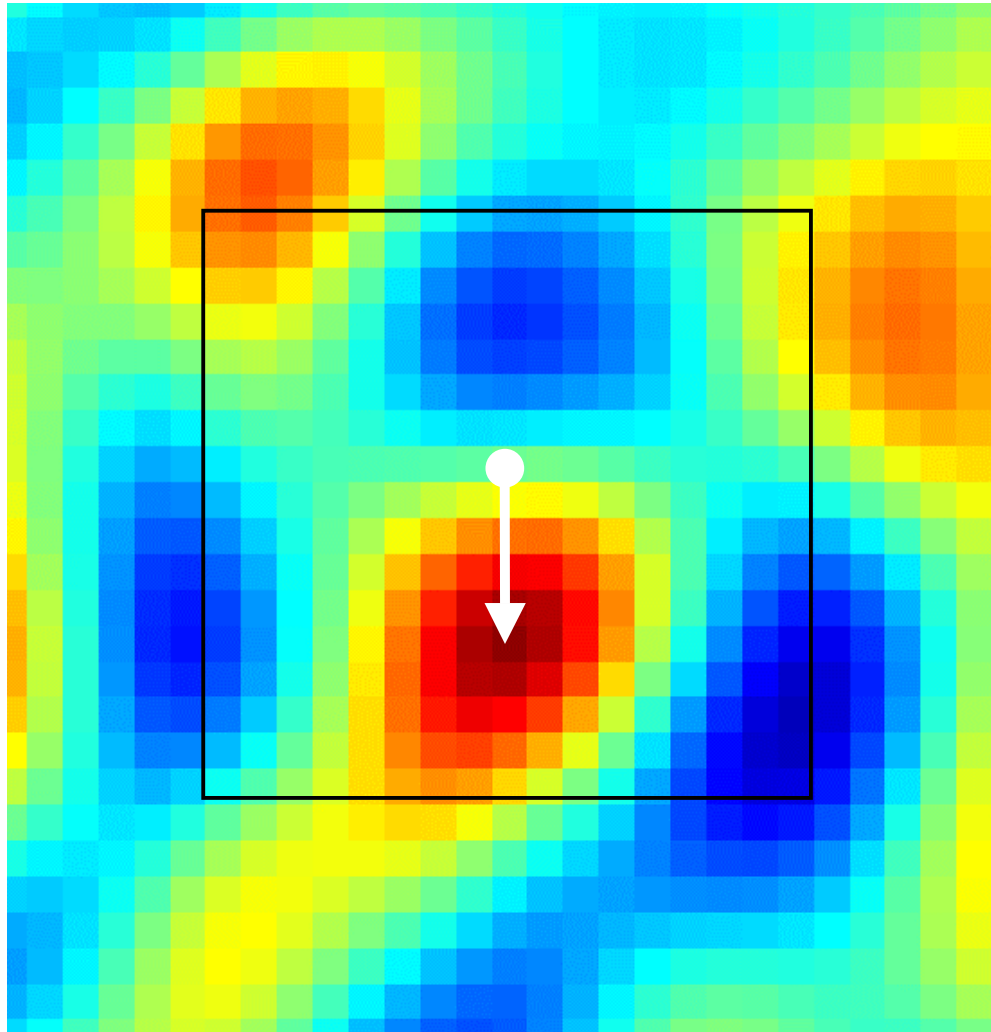
- Take a small patch, p from image n and cross-correlate it with image $n+k$.
- Find position of maximum.
- Distance from origin of p to maximum is the best fit for the displacement in time k .
- Pick new patch and repeat until all patches from frame n are found.
- Go to frame $n+1$ and repeat until all frames are done.

PIV

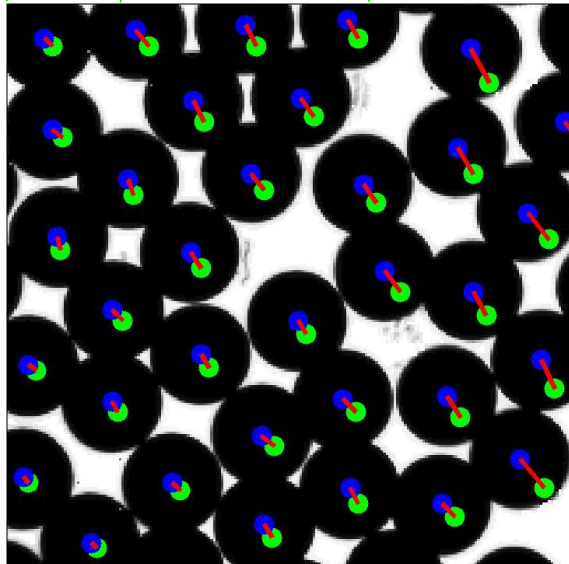
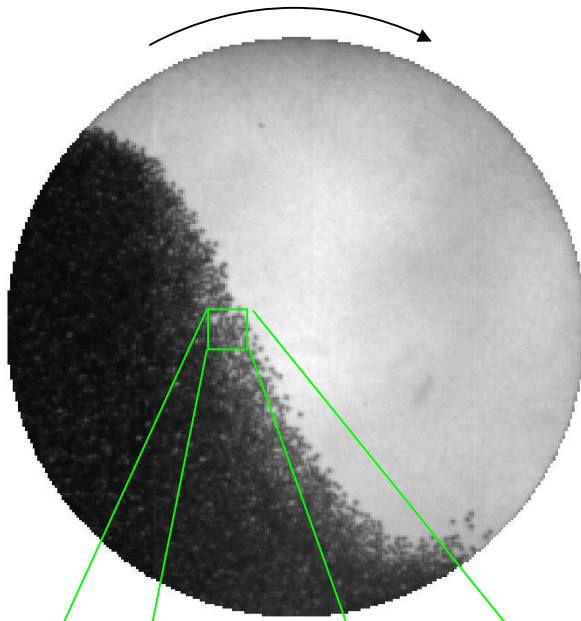


Correlation

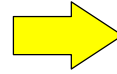
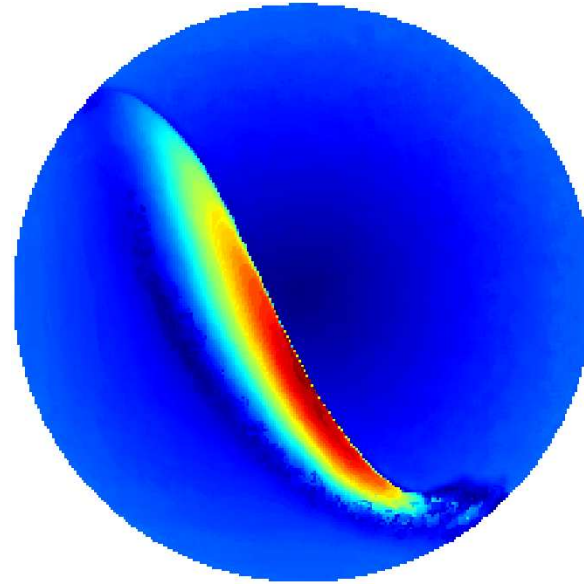
PIV



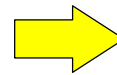
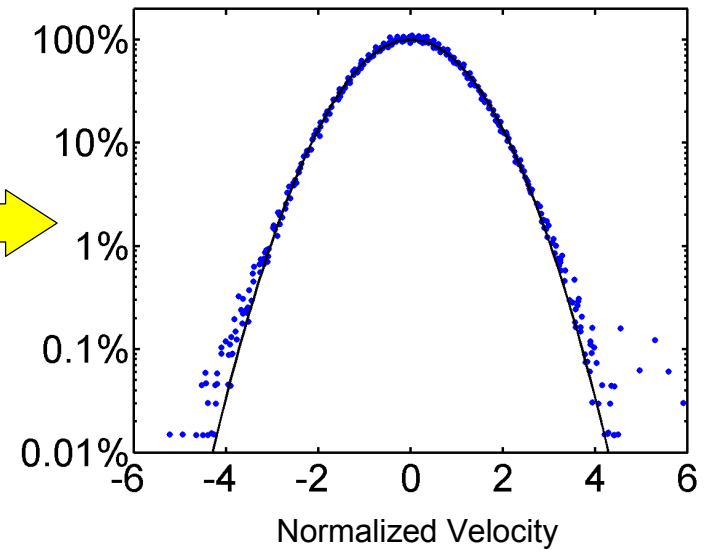
Flow snapshot



Speed



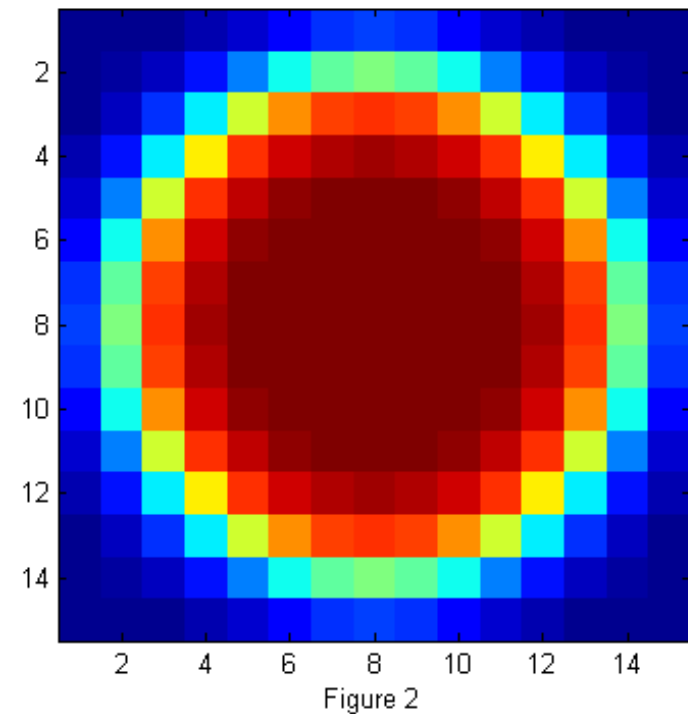
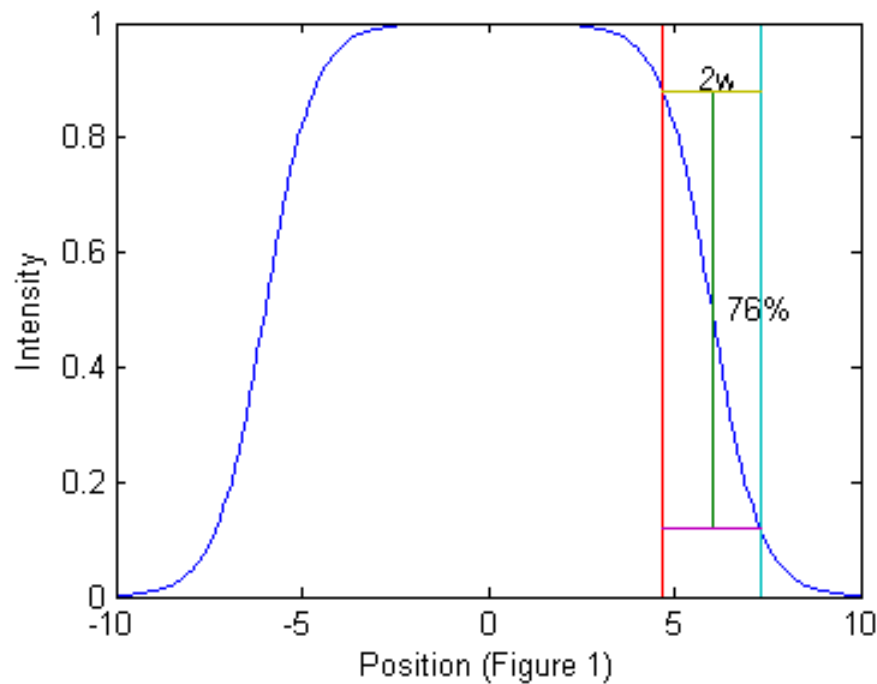
Velocity Histogram



Particle Tracking

$$I_c(\vec{x}) = \sum_{n=1}^N I_p(\vec{x} - \vec{x}_n(t); D, \dots),$$

$$I_p(\vec{x}; D, w) = [1 - \tanh(\frac{|\vec{x}| - D/2}{w})]/2.$$



Particle Tracking

$$\chi^2(\vec{x}_0; D, w) = \int W(\vec{x} - \vec{x}_0) [I(\vec{x}) - I_p(\vec{x} - \vec{x}_0; D, w)]^2 d\vec{x},$$

$$\chi^2(\vec{x}_0; D, w) = \int W(\vec{x} - \vec{x}_0) [I(\vec{x})^2 - 2I(\vec{x})I_p(\vec{x} - \vec{x}_0; D, w) + I_p(\vec{x} - \vec{x}_0; D, w)^2] d\vec{x},$$

$$\chi^2(\vec{x}_0; D, w) = I^2 \otimes W - 2I \otimes (WI_p) + \langle WI_p^2 \rangle,$$

$$W = 1; \quad \chi^2(\vec{x}_0; D, w) = \int I^2 d\vec{x} - 2I \otimes I_p + \langle I_p^2 \rangle,$$

$$W = I_p; \quad \chi^2(\vec{x}_0; D, w) = I^2 \otimes I_p - 2I \otimes I_p^2 + \langle I_p^3 \rangle.$$

Particle Tracking

$$W = I_p; \quad \chi^2(\vec{x}_0; D, w) = I^2 \otimes I_p - 2I \otimes I_p^2 + \langle I_p^3 \rangle.$$

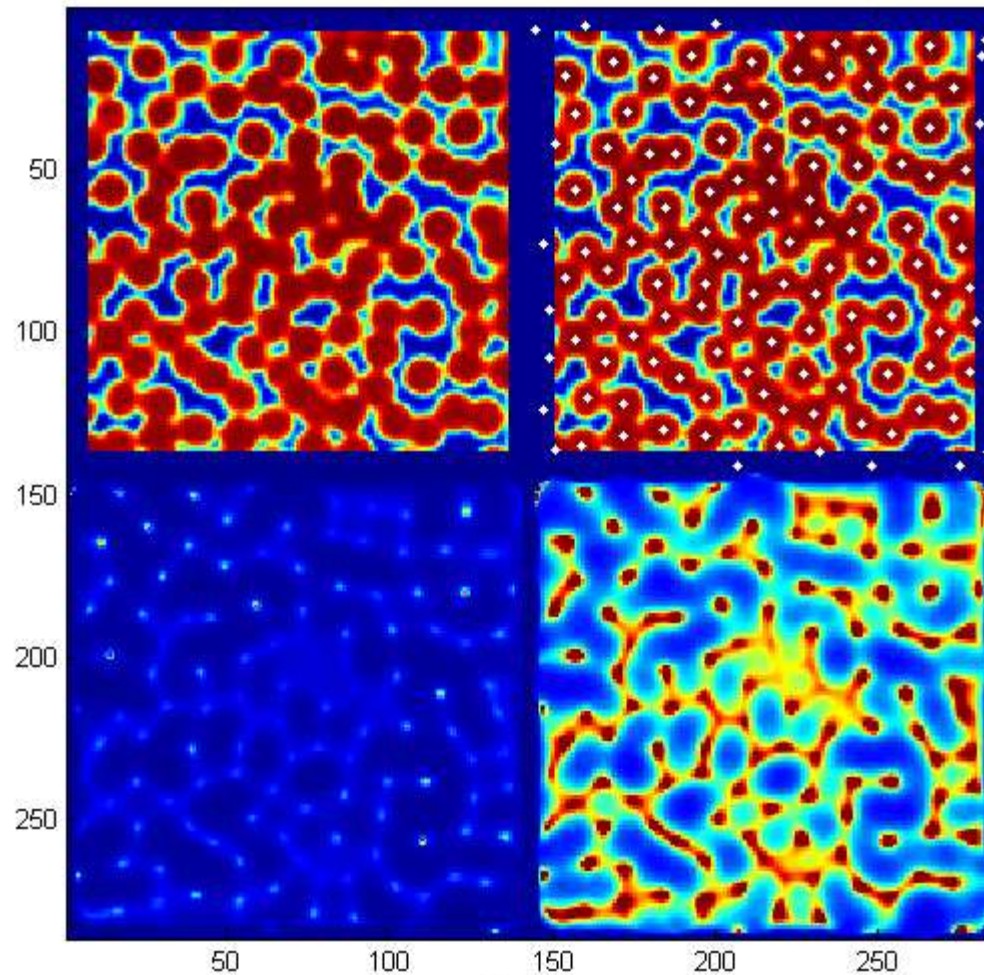


Figure 3.

Particle Tracking

$$\chi^2(\vec{x}_n; D, w) = \int [I(\vec{x}) - I_c(\vec{x}, \vec{x}_n)]^2 d\vec{x},$$

$$I_c(\vec{x}, \vec{x}_n) = \sum_n W_n(\vec{x}) I_p(\vec{x} - \vec{x}_n; D, w),$$

Real, Calculated, and χ^2 images

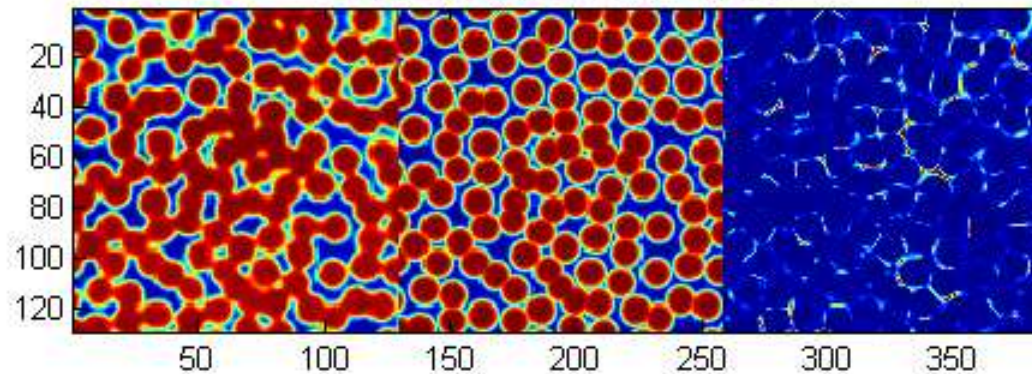


figure 4.

Particle Tracking

$$\frac{\partial \chi^2(\vec{x}_n^*; D, w)}{\partial \vec{x}_n^*} = 0,$$

New $\chi^2=167.01$

Original $\chi^2=228.16$

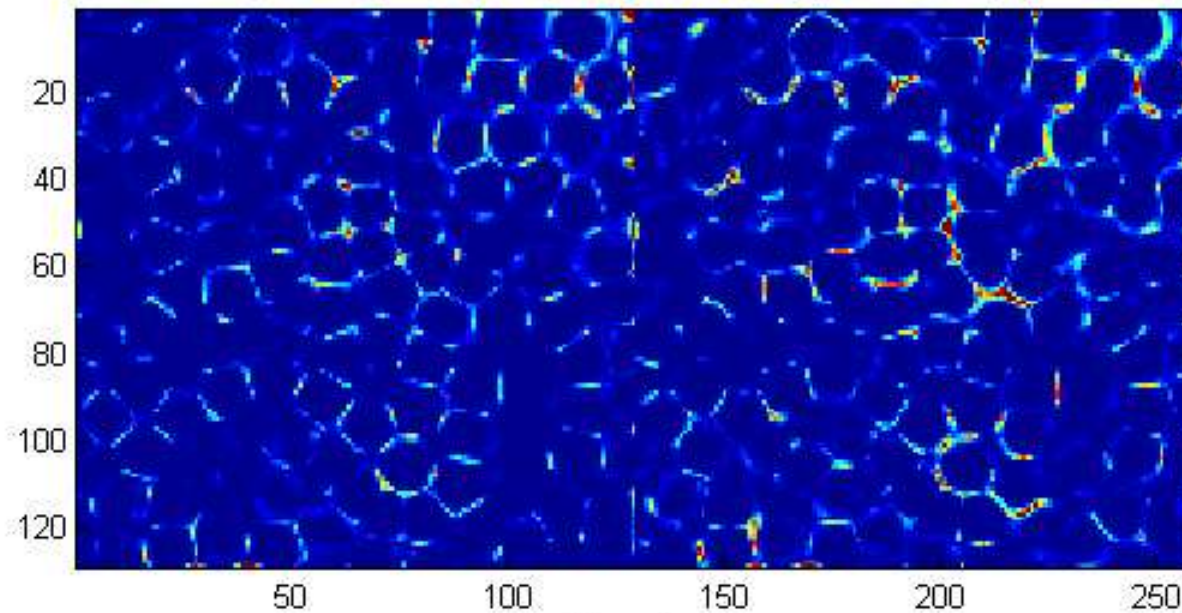


figure 5.

Particle Tracking

$$\frac{\partial \chi^2(\vec{x}_0; D^*, w^*)}{\partial D^*} = 0; \quad \frac{\partial \chi^2(\vec{x}_0; D^*, w^*)}{\partial w^*} = 0$$

New $\chi^2 = 87.72$

Original $\chi^2 = 228.16$

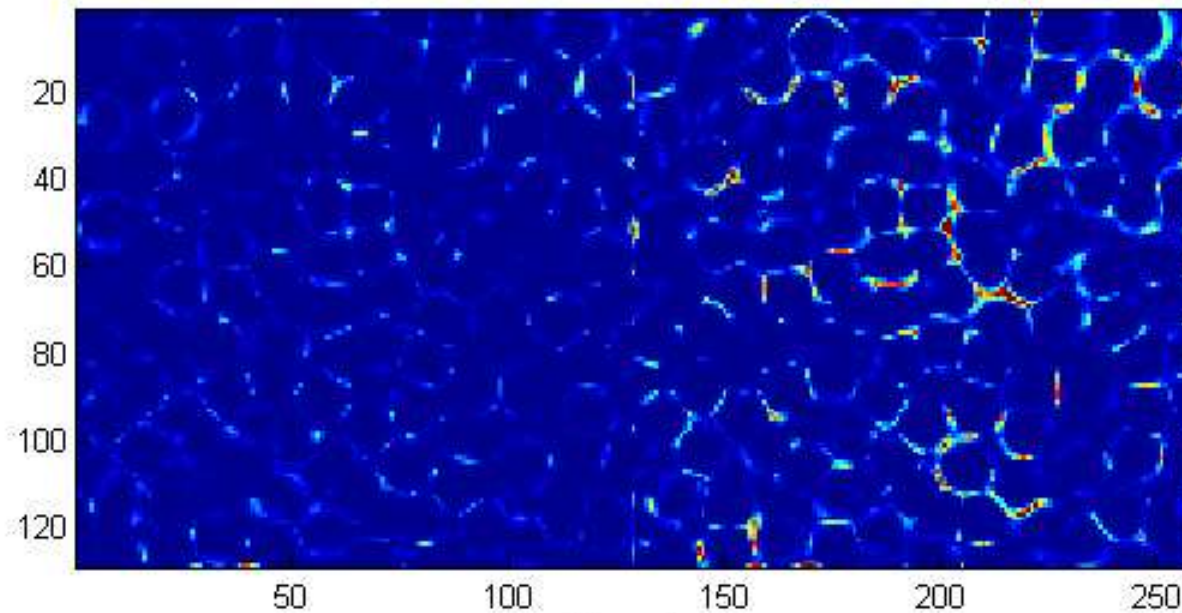


figure 6.

Particle Tracking

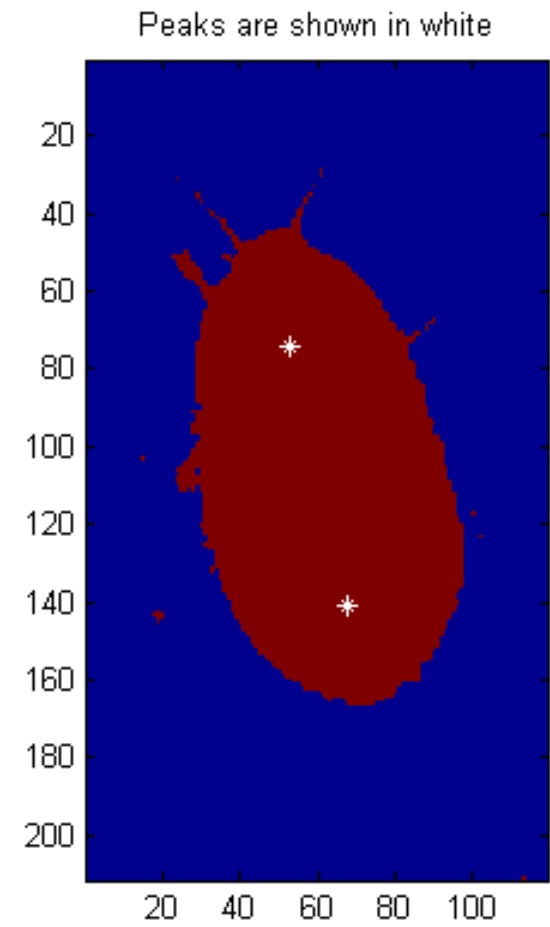
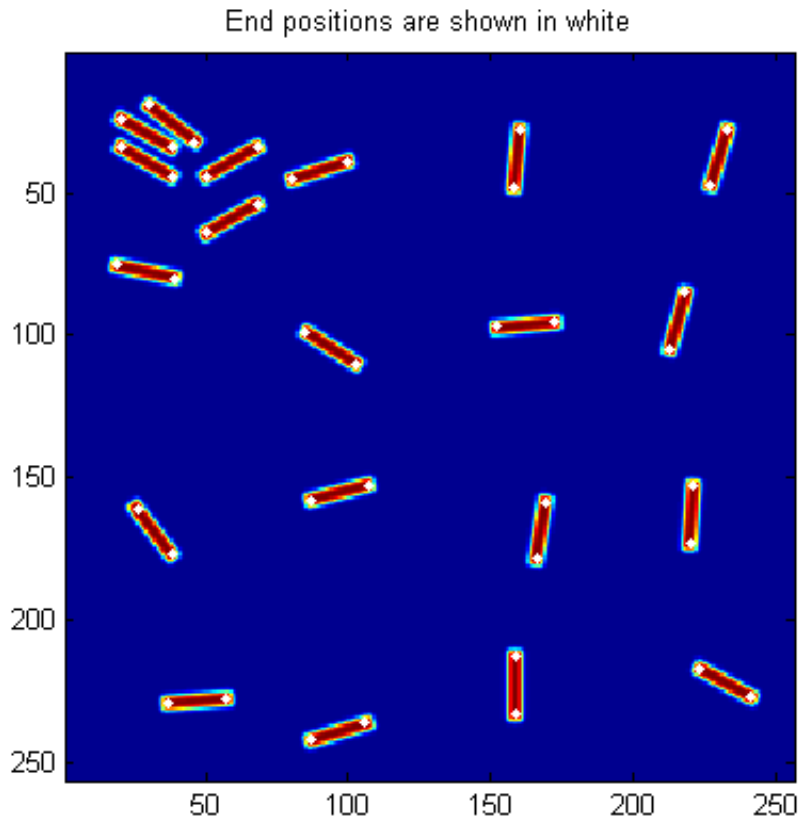
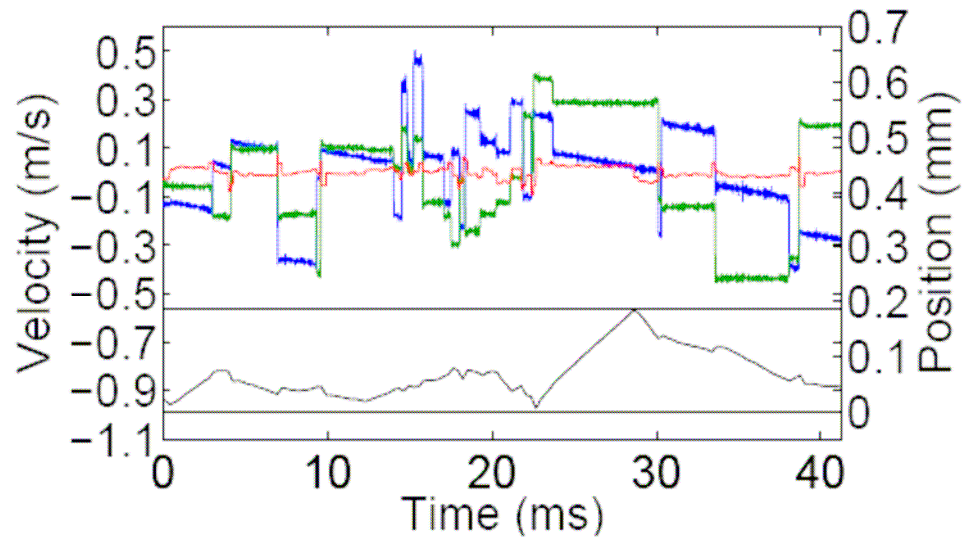
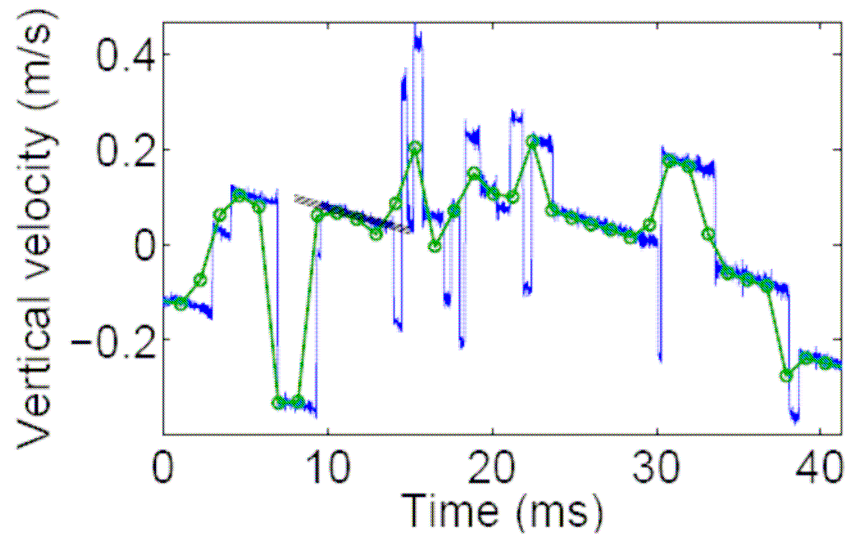


Figure 13.

Particle Tracking



The End

