Structures in Granular Gases

Computer Experiments meet Kinetic Theory

Thorsten Pöschel - University Erlangen-Nuremberg

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Outline:

- What is a Granular Gas? What is Kinetic Theory?
  Why comupter experiments?
- pair collisions
- velocity distribution function
- correlations

What is a Granular Gas?

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It is the same as a molecular gas **but** the particles interact dissipatively.

Some implications:

- dilute system (mean free path >> particle size)
- no correlations (Molecular Chaos, *see later*)
- particles are **macroscopic bodies**

dissipation = distribution of energy of motion to many internal degrees of freedom

typically d=μm … cm … m

What is a Granular Gas?

Saturn, seen by Cassini NASA web site

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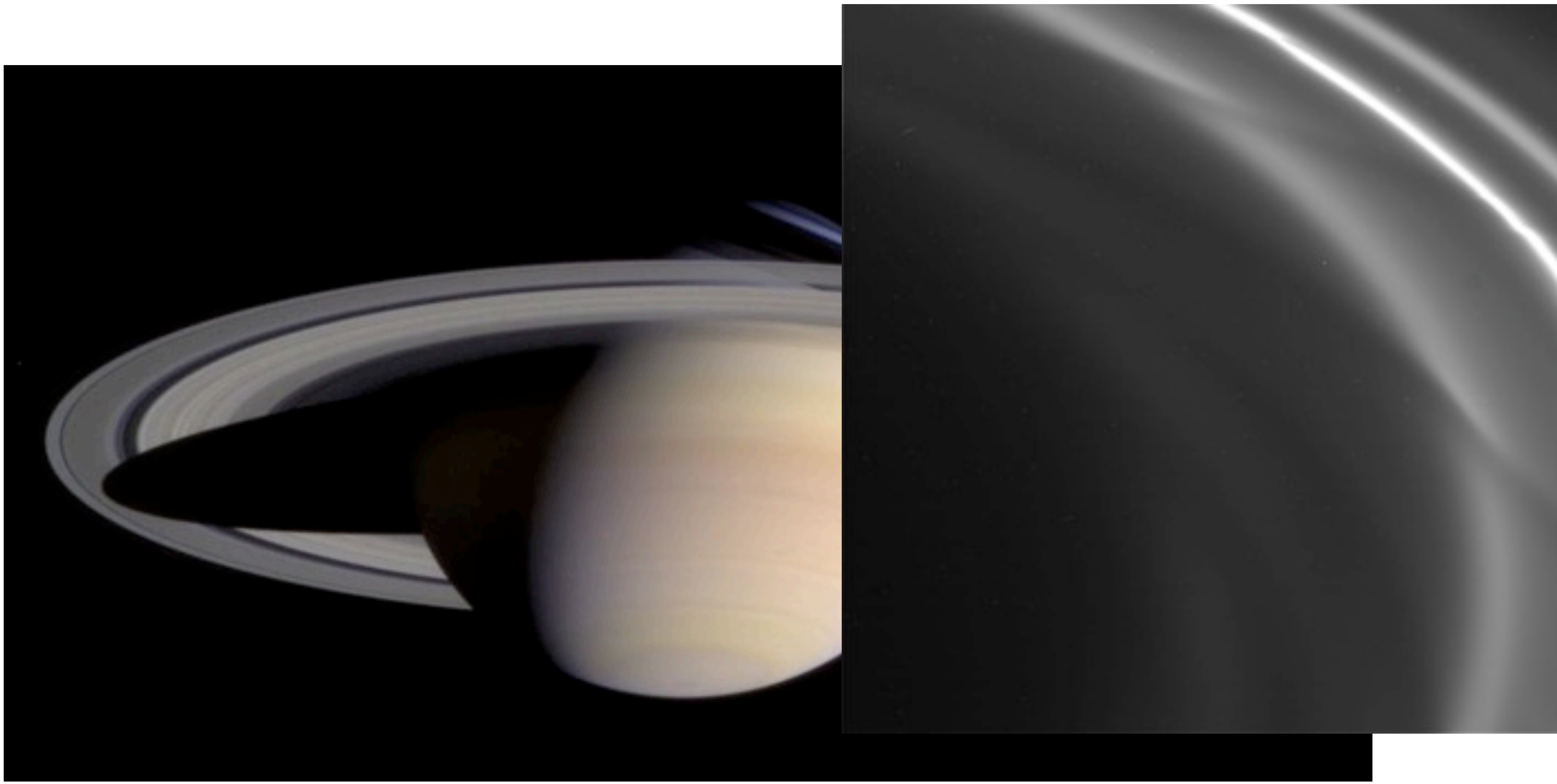

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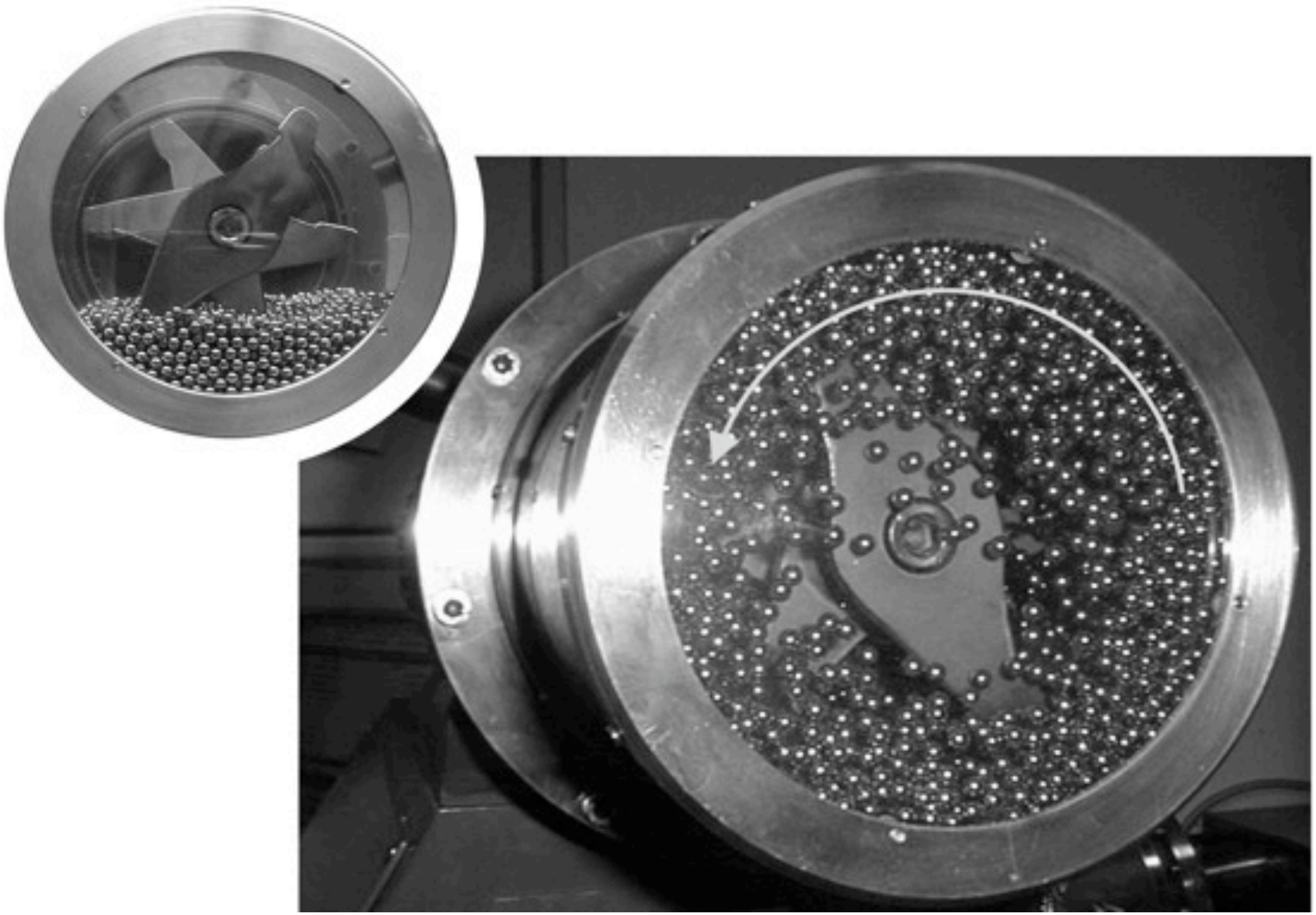

Reichhard'04

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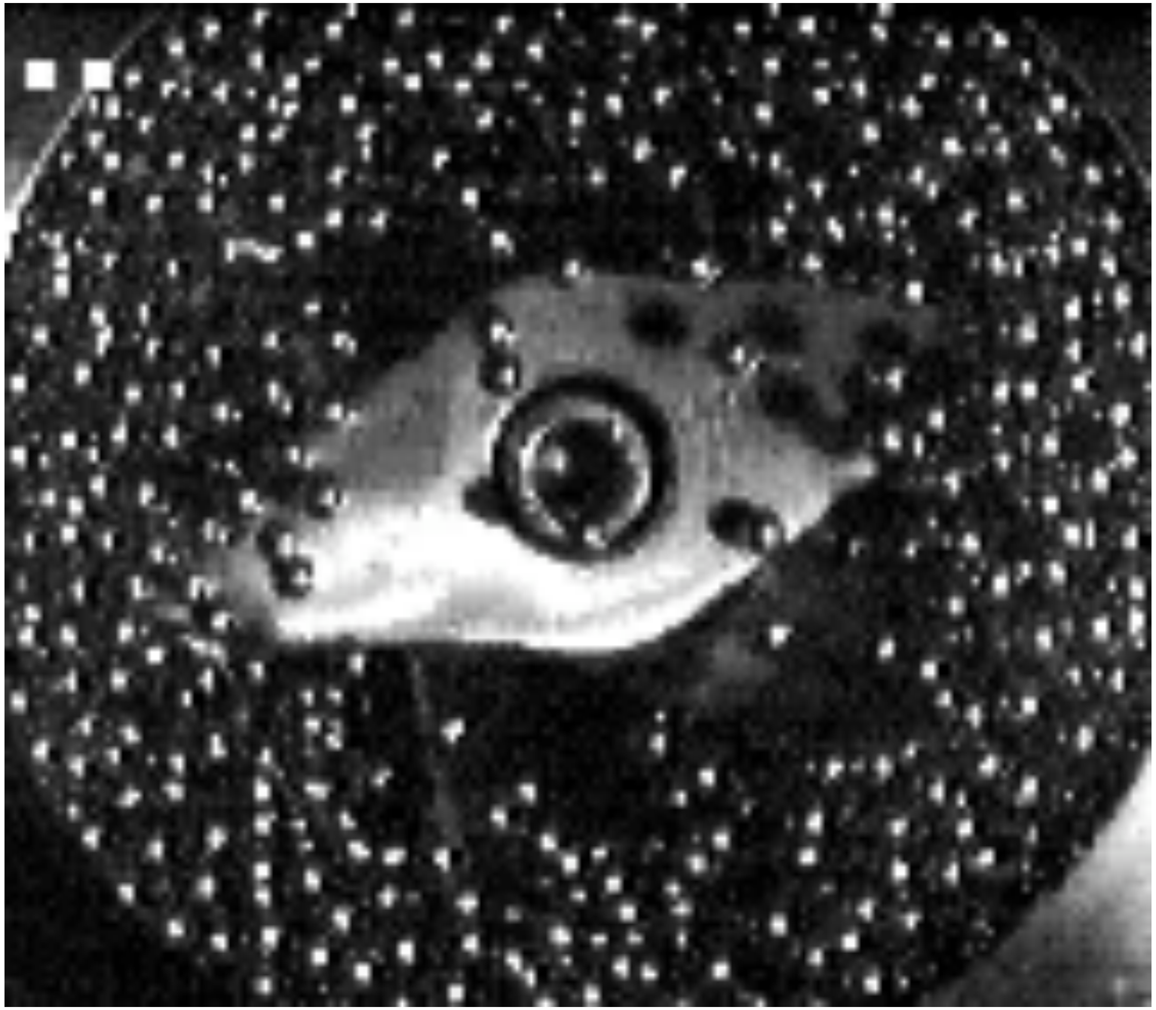

Reichhard'04

What is Kinetic Theory?

KT explains macroscopic properties of gases (such as pressure, temperature, …) by considering their molecular composition and motion.

D. Bernoulli (1738)
R. Claudius (1857)
J. Maxwell (1859)
L. Boltzmann (1871)
A. Einstein (1905)
M. Smoluchowski (1906)

Multi scale problem:

particle material and surfaces (nm...µm)
→ particle collision properties (mm...cm)
→ spatial structures (cm...km)
→ hydrodynamics (up to very large)

Why computer experiments?

phenomena depend sensitively on the boundary conditions and on external force fields (gravity).

solution: 2d (no motion due to gravity but rolling
= correlation between linear and rotational motion)

solution: air table (aerodynamic effects)

solution: slight vertical vibration (uncontrolled supply of energy)

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Granular Gas experiments are not easy … but possible.

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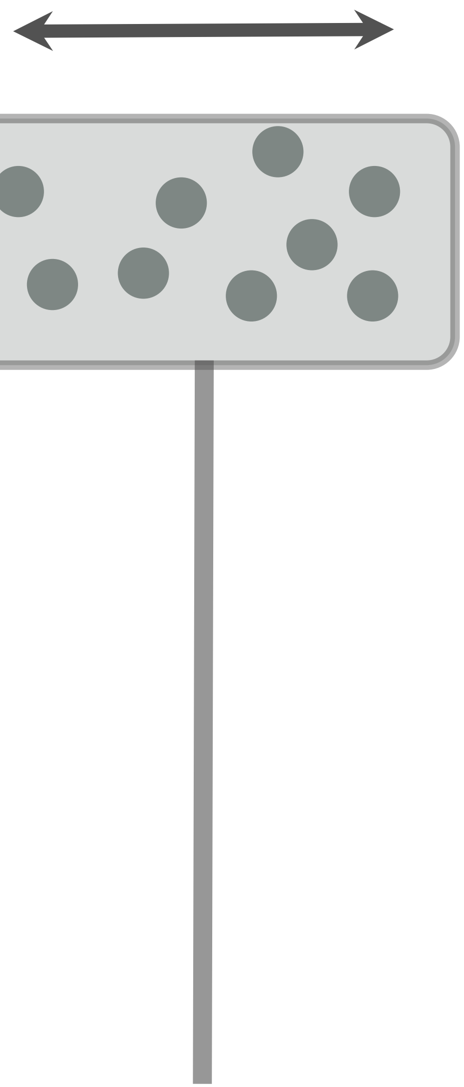

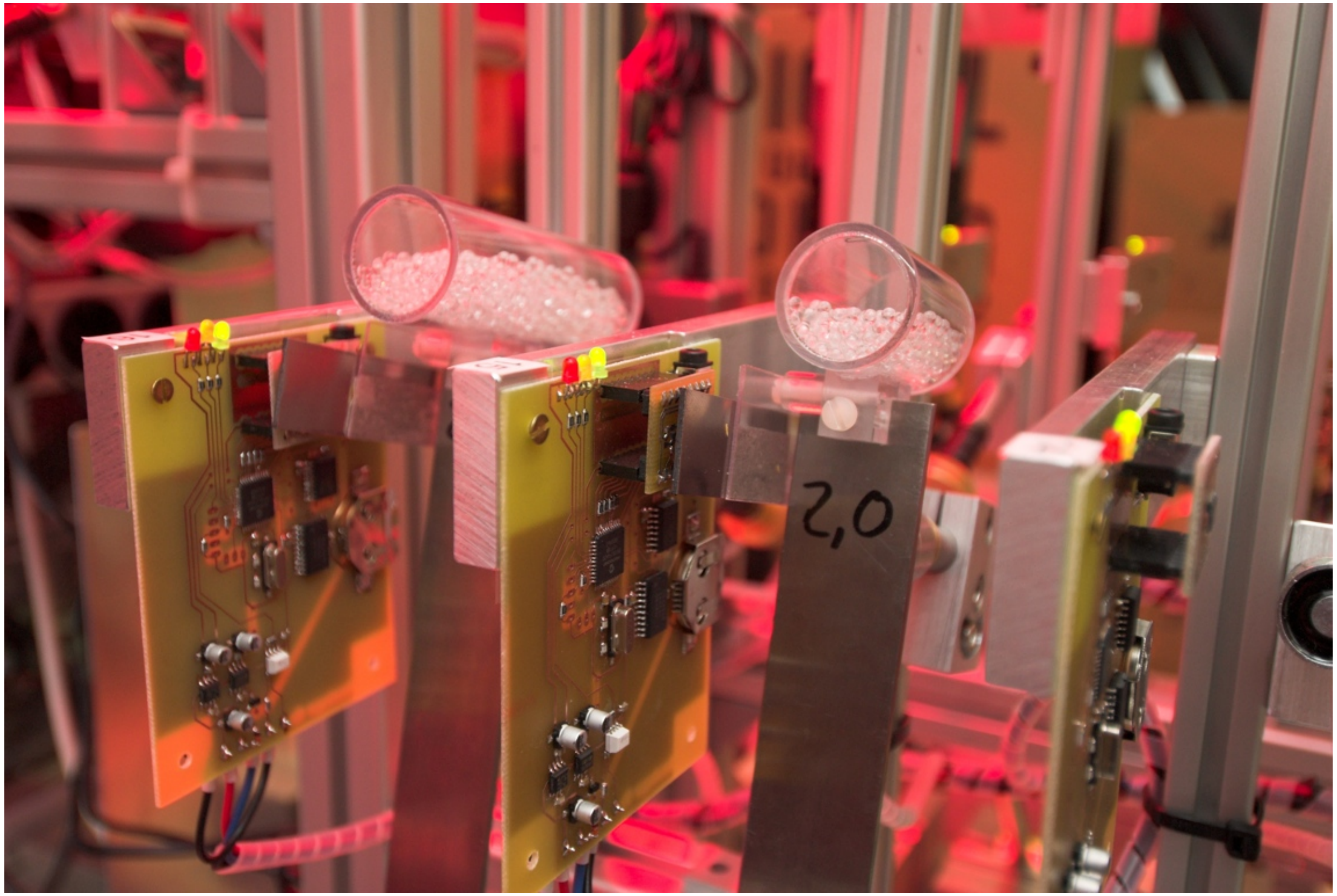

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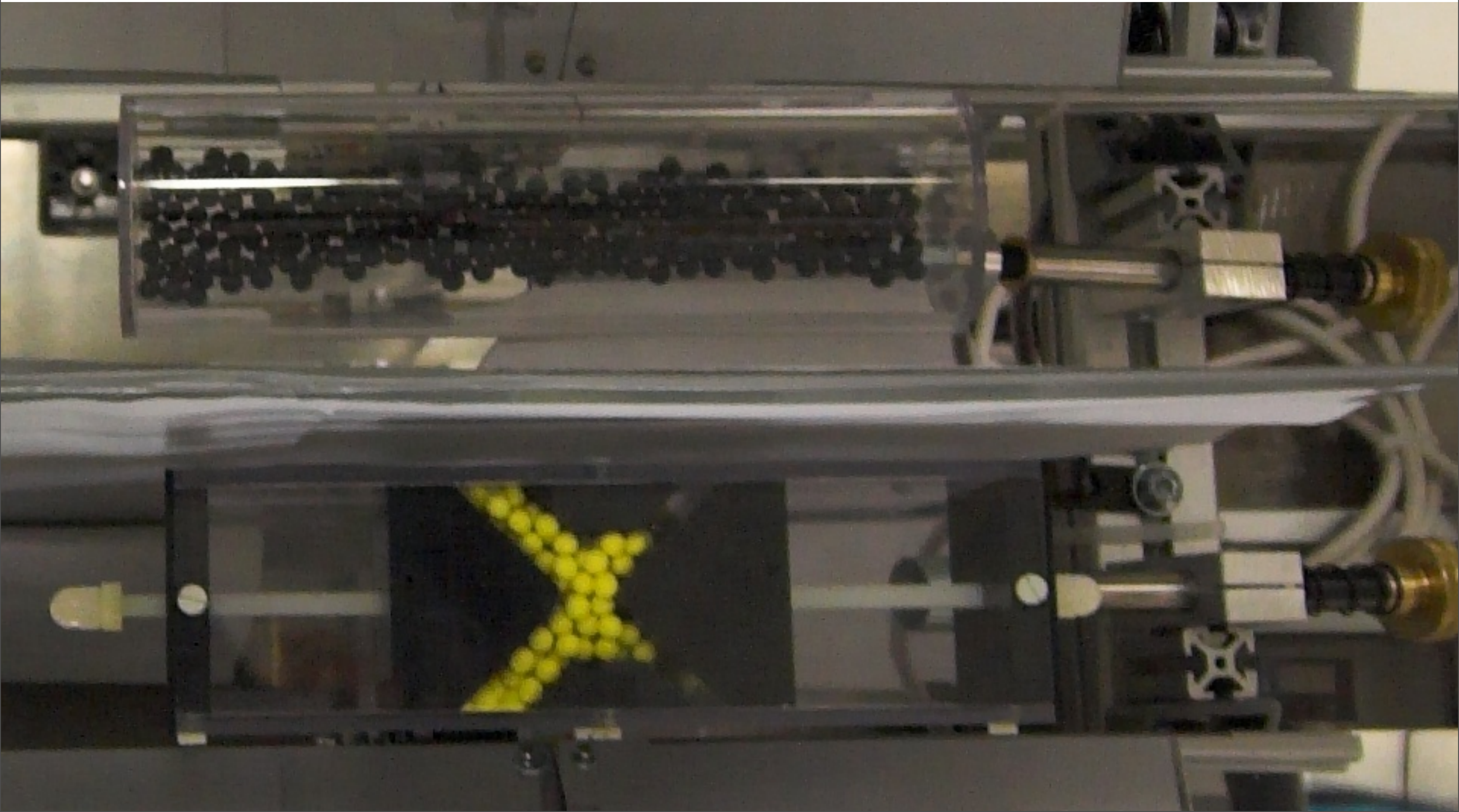

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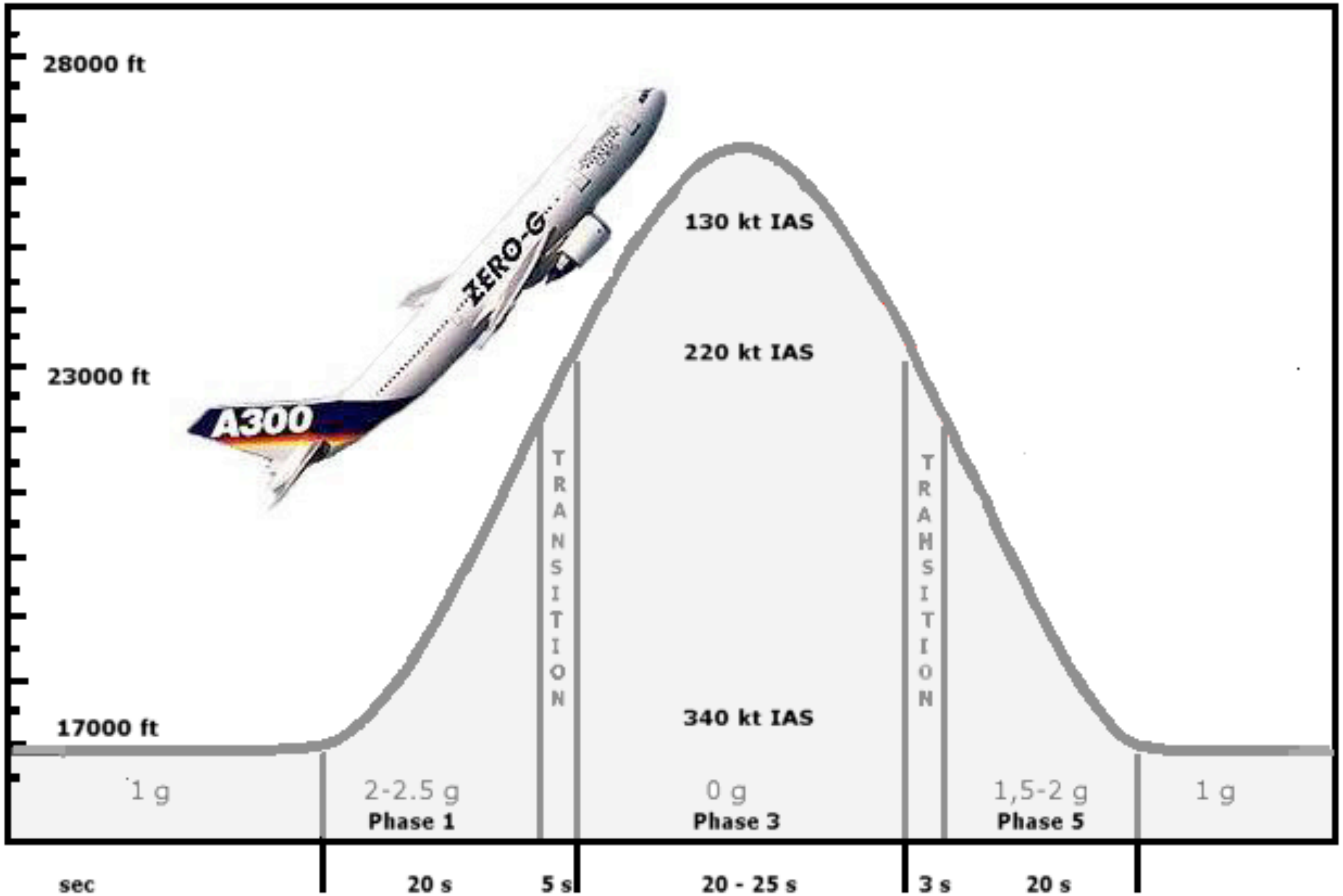

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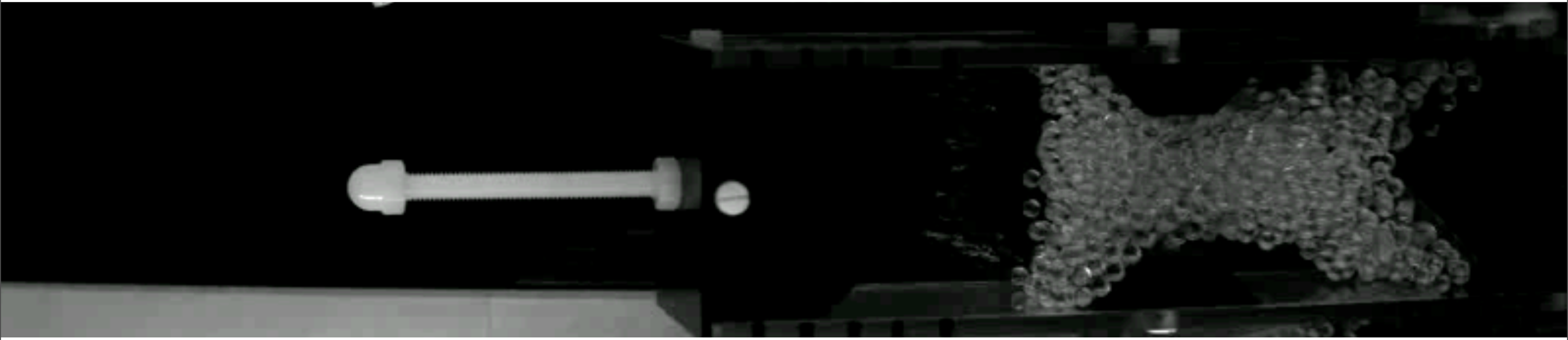
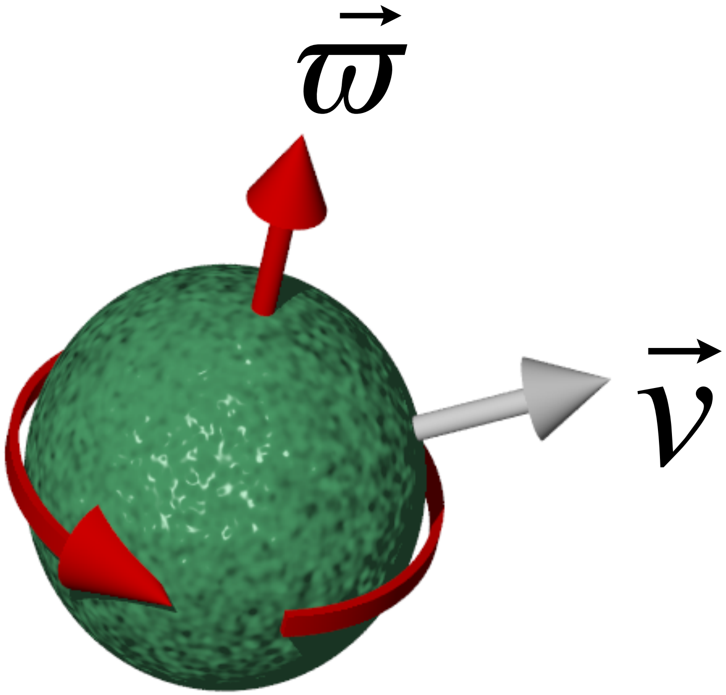


4mm glass beads, A=8 cm, 500 fps
no gravity !!

particle collisions

Forces and torques are model specific. Here: rough spheres.

$$\vec{F}_{ij} = \left(\vec{F}_{ij} \cdot \vec{e}_{ij}^{\,n}\right) \vec{e}_{ij}^{\,n} + \left[\vec{F}_{ij} - \left(\vec{F}_{ij} \cdot \vec{e}_{ij}^{\,n}\right) \vec{e}_{ij}^{\,n}\right] = F_{ij}^{n} \vec{e}_{ij}^{\,n} + F_{ij}^{t} \vec{e}_{ij}^{\,t}$$



normal force:
Hertz contact force
(bulk material property)

tangential force:
several models
(material and surface property)

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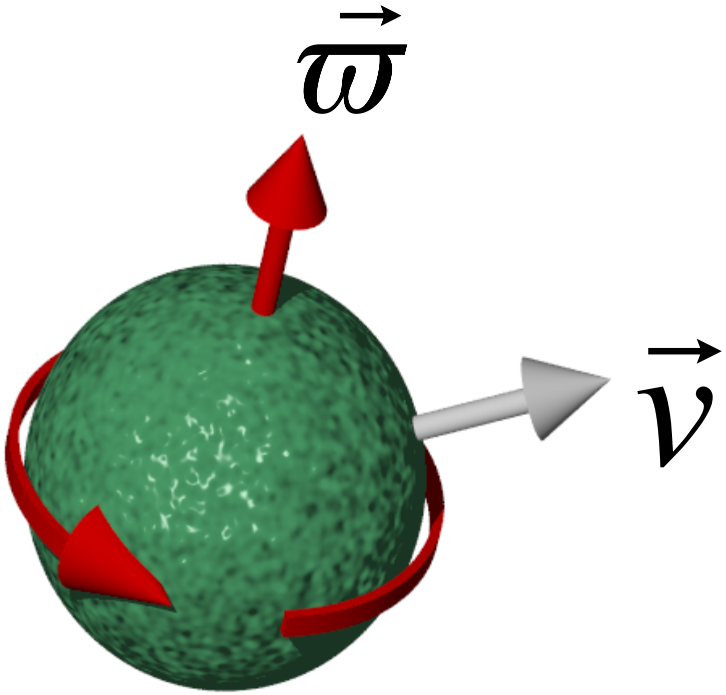

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Hertz contact force
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unit vectors

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several models
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particle collisions

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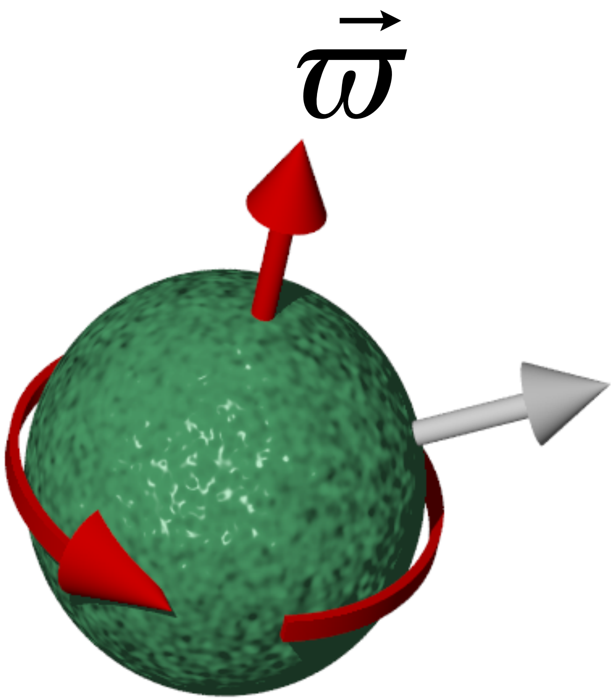

possible method to obtain the dynamics:
Molecular Dynamics
= integrate Newton's equation of motion
for all N particles

N may be large !!! (millions of coupled PDE's)
Observation time may be long !!!

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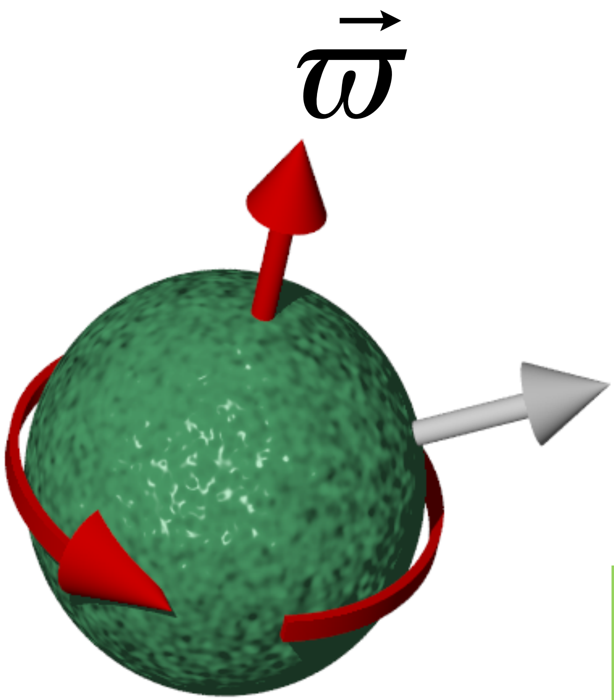

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Can we describe the dynamics of the N-particle system as a sequence of 2-particle collisions?

Idea: replace integration of Newton's equation by a collision rule

$$\vec{g} \equiv \vec{v}_1 - \vec{v}_2 \qquad \vec{e} \equiv \left(\vec{r}_1 - \vec{r}_2\right) / \left|\vec{r}_1 - \vec{r}_2\right|$$

$$(\vec{g} \cdot \vec{e})' = -\,\varepsilon_n\, \vec{g} \cdot \vec{e}$$

$$\varepsilon_n = \begin{cases} 1: & \textit{elastic collision} \\ 0: & \textit{perfectly inelastic collision} \end{cases}$$

coefficient of restitution is the central quantity in the kinetic theory of granular gases.

total momentum conserved

$$\vec{v}\,'_1 = \vec{v}_1 - \frac{1+\varepsilon}{2}\,\vec{g} \qquad \vec{v}\,'_2 = \vec{v}_2 + \frac{1+\varepsilon}{2}\,\vec{g}$$

collision law

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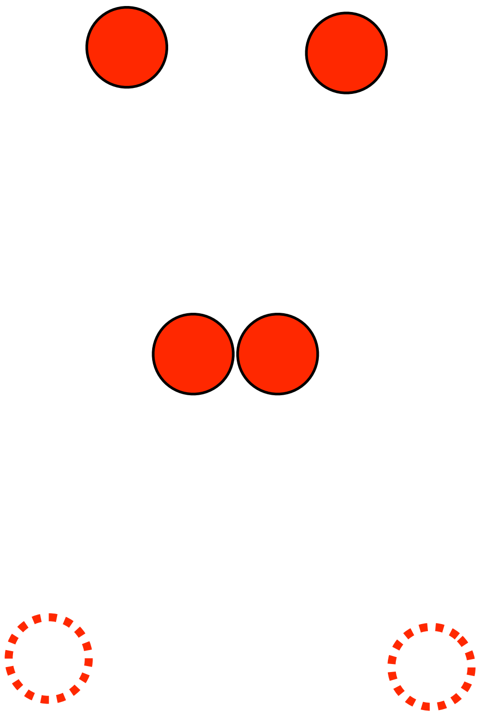

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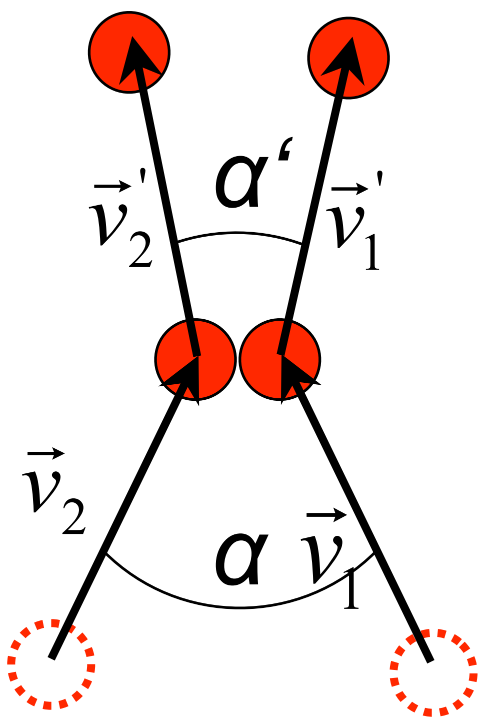

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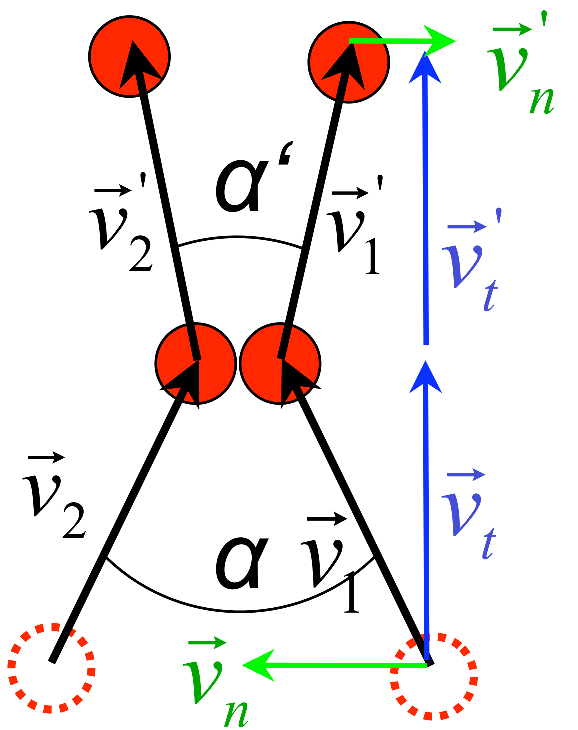

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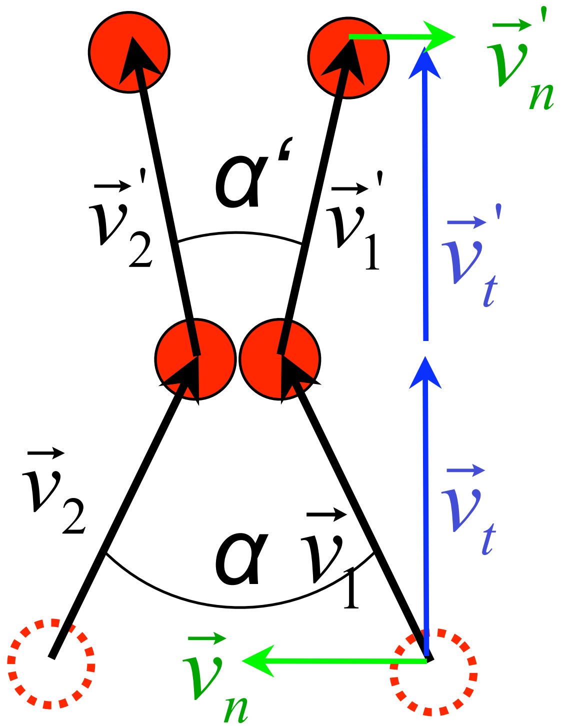

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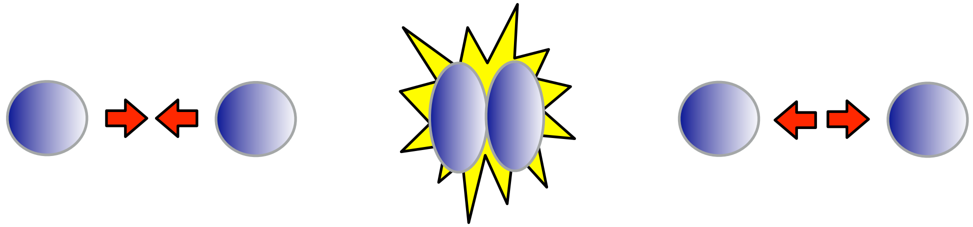
basic algorithm (event driven MD)

1. determine time of the next collision
2. compute coordinates for this time
3. compute new velocities for colliders
4. record data
5. continue with next collision

extremely inefficient: **$O(N^2) \rightarrow O(N \log N)$**

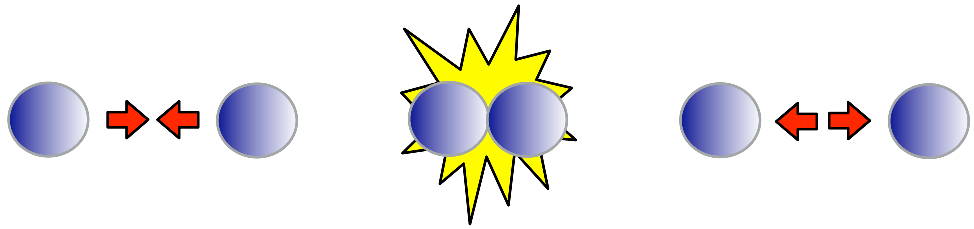
…. → **$O(N)$**

soft sphere model
➔ MD (forces)



$\vec{F}_{ij}(\ldots)$ characterizes collision

hard sphere model
➔ event driven MD



ε characterizes collision

Idea: replace forces by coefficients of restitution

$$\left(\vec{g}_{ij}^{\,n}\right)' = -\varepsilon^{n}\,\vec{g}_{ij}^{\,n} \qquad\qquad 0 \le \varepsilon^{n} \le 1$$

with

$$\left(\vec{g}_{ij}^{\,t}\right)' = \varepsilon^{t}\,\vec{g}_{ij}^{\,t} \qquad\qquad -1 \le \varepsilon^{t} \le 1$$

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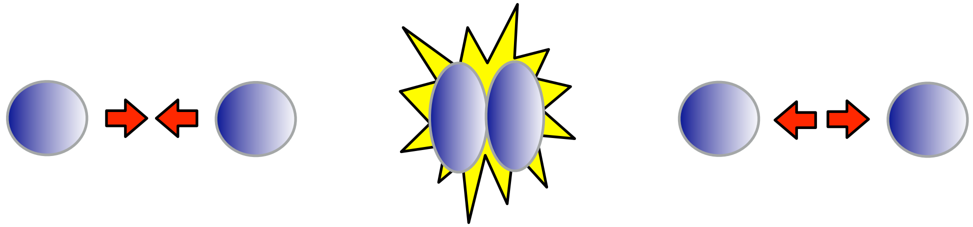

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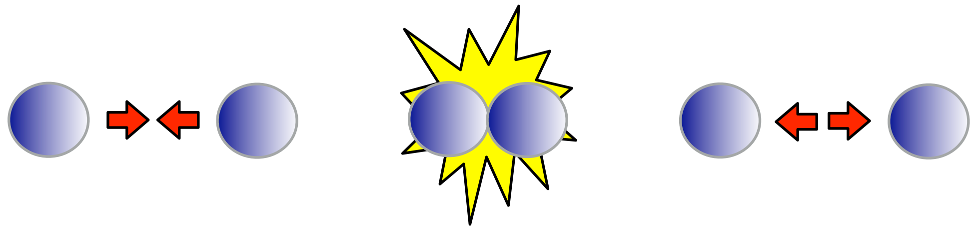

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not perfectly correct (almost)

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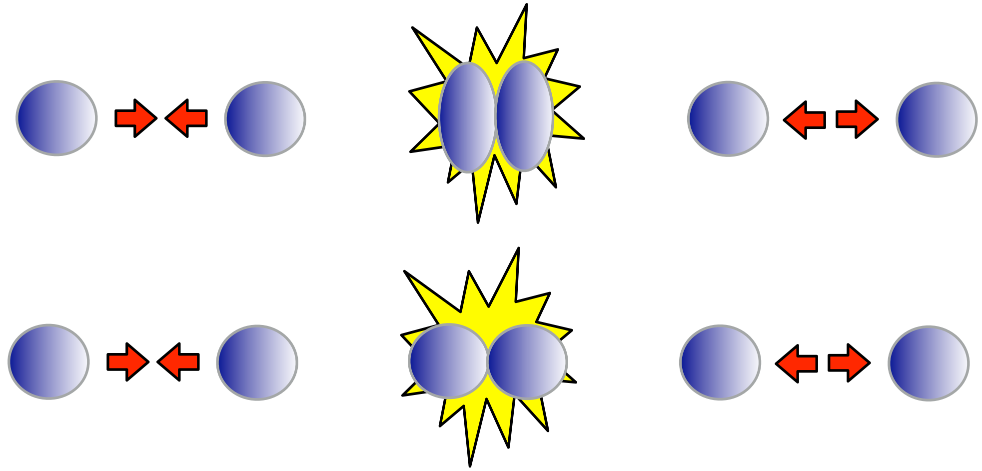

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with

$$0 \le \varepsilon^{n} \le 1 \qquad -1 \le \varepsilon^{t} \le 1$$

not perfectly correct (almost)

- hard sphere model is a great simplification:

$$\int_{t_{\mathrm{start}}}^{t_{\mathrm{end}}} \vec{F}_{ij}\left(\vec{v}_i, \vec{v}_k, \vec{\omega}_i, \vec{\omega}_j\right) \ldots dt \rightarrow \left[\varepsilon^{n}\left(g^{n}\right), \varepsilon^{t}\left(g^{n},\, g^{t}\right)\right]$$

- ε(...) must be derived from the interaction force
- valid only for low density (no 3-particle contacts allowed)

derivation of coefficients of restitution from interaction forces

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…. is hard work!

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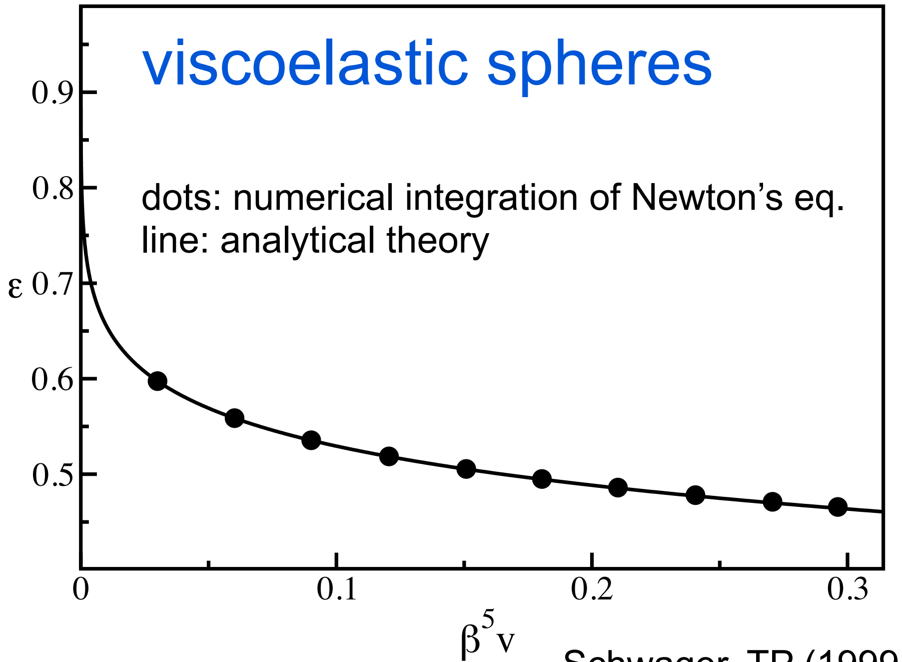
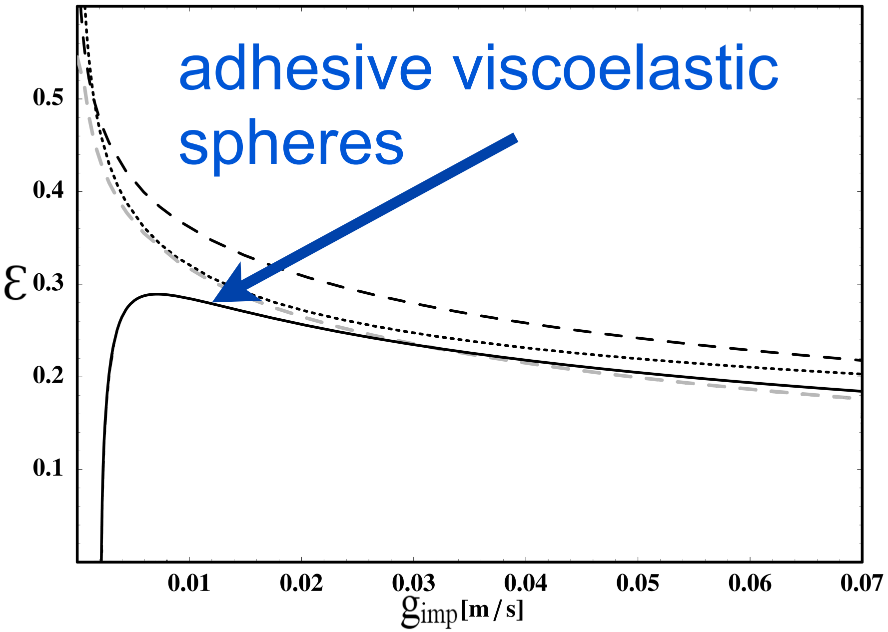


Schwager, TP (1999,2008)

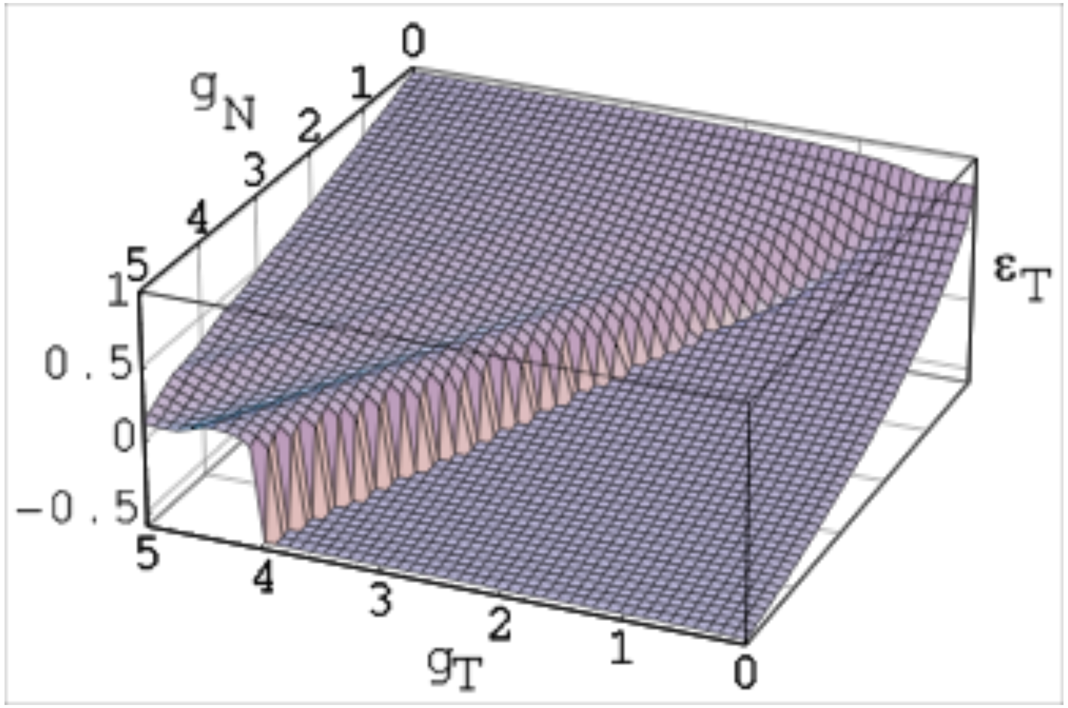


Brilliantov, Albers, Spahn, TP (2008)

$$\varepsilon = 1 + \sum_{k=0}^{\infty} a_k \, v^{k/10}$$

derivation of coefficients of restitution from interaction forces

…. is hard work!



Brilliantov, Spahn, Hertzsch, Pöschel'96

tangential coefficient of restitution for viscoelastic spheres

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coefficients of restitution can be measured even if the forces are unknown!

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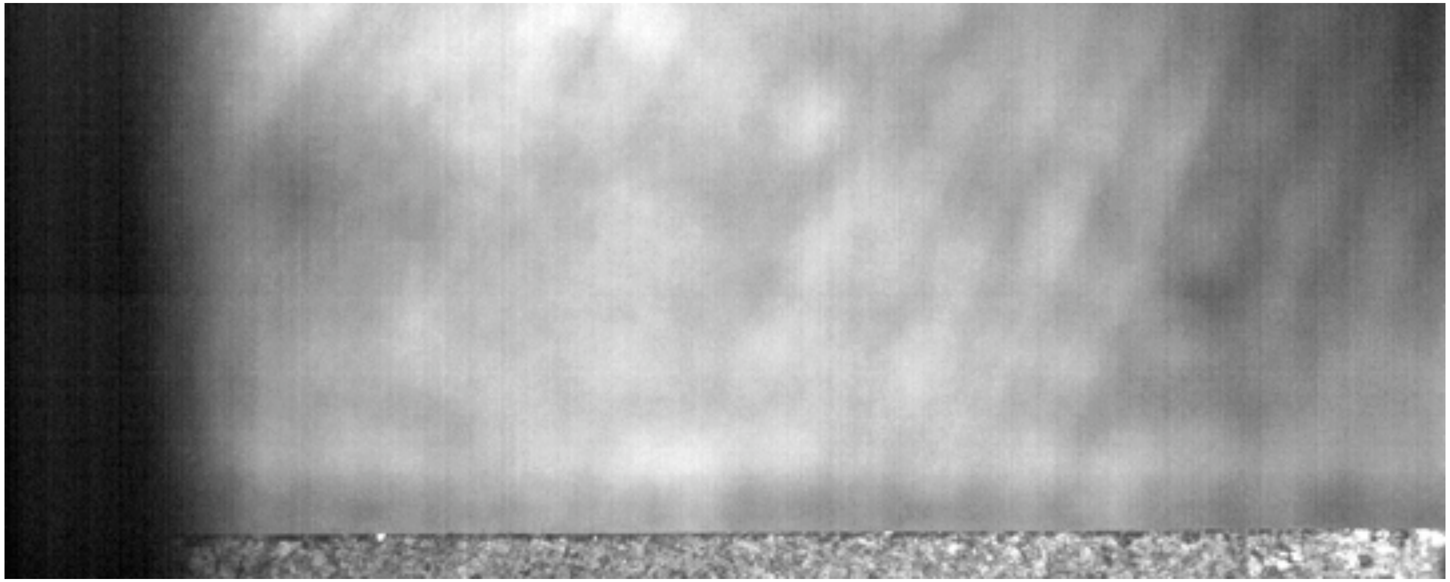

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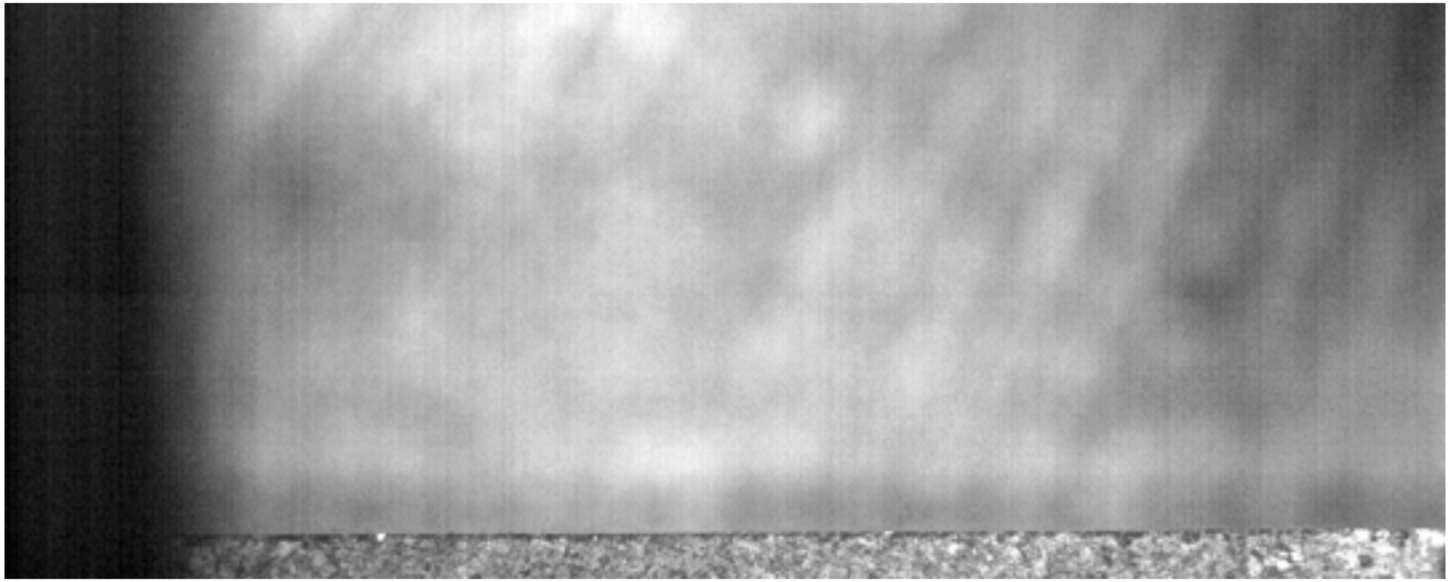
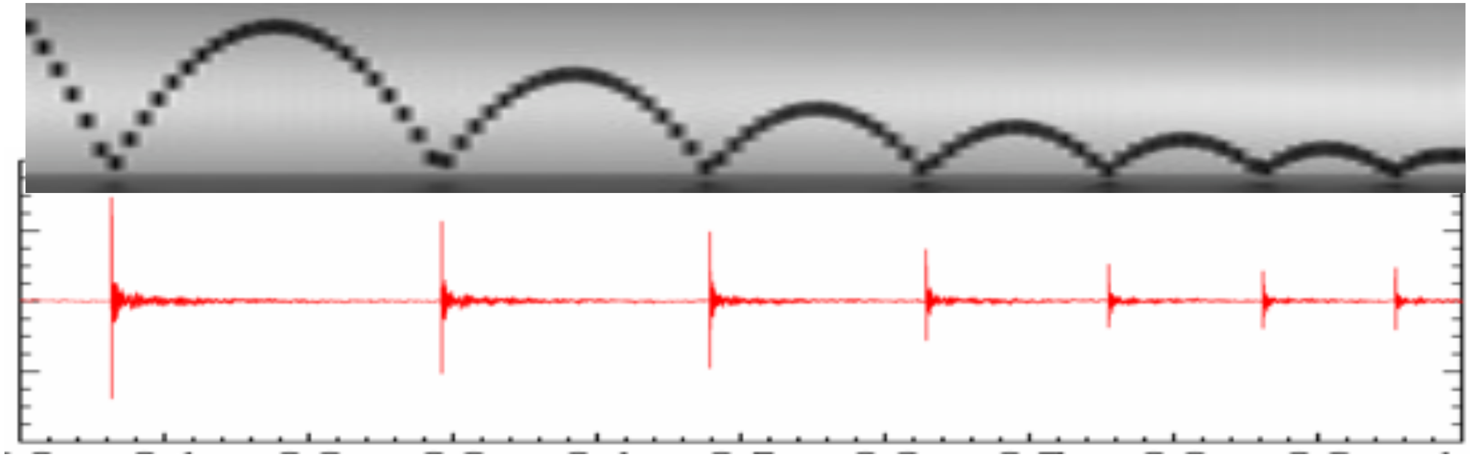
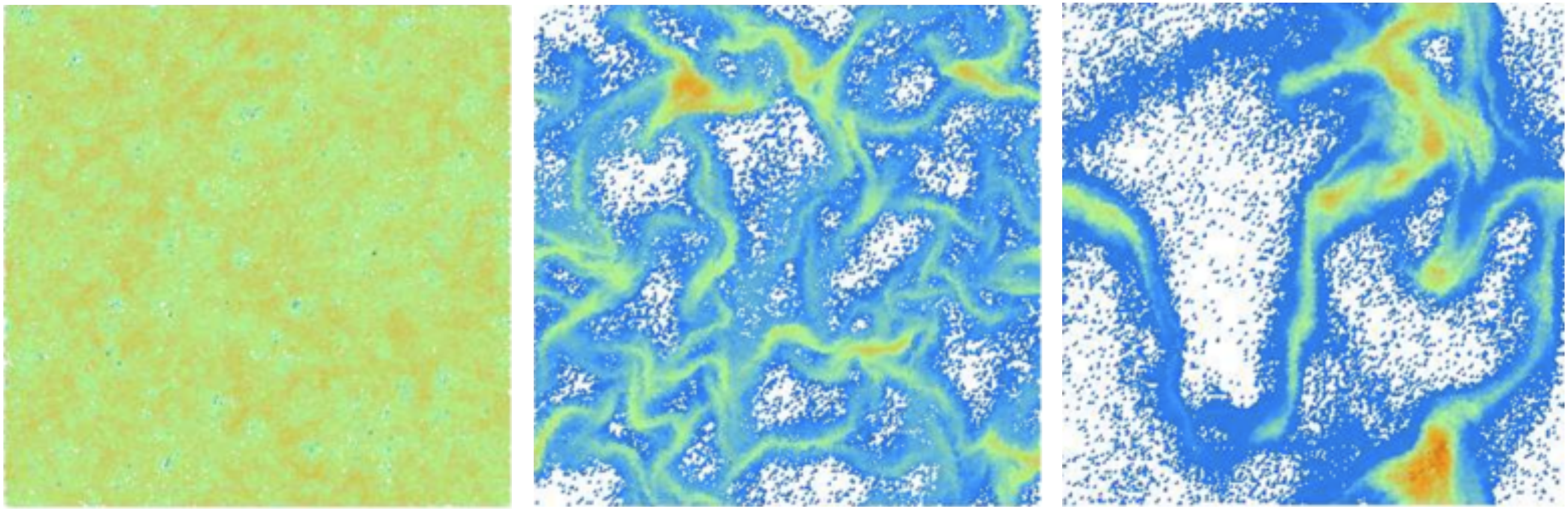




$$\varepsilon_i^n\left(g_i^n\right)=-\frac{t_{i+1}-t_i}{t_i-t_{i-1}}$$

$$g_i^n=-\frac{G}{2}\left(t_i-t_{i-1}\right)$$

Event-driven MD: force-free granular gas



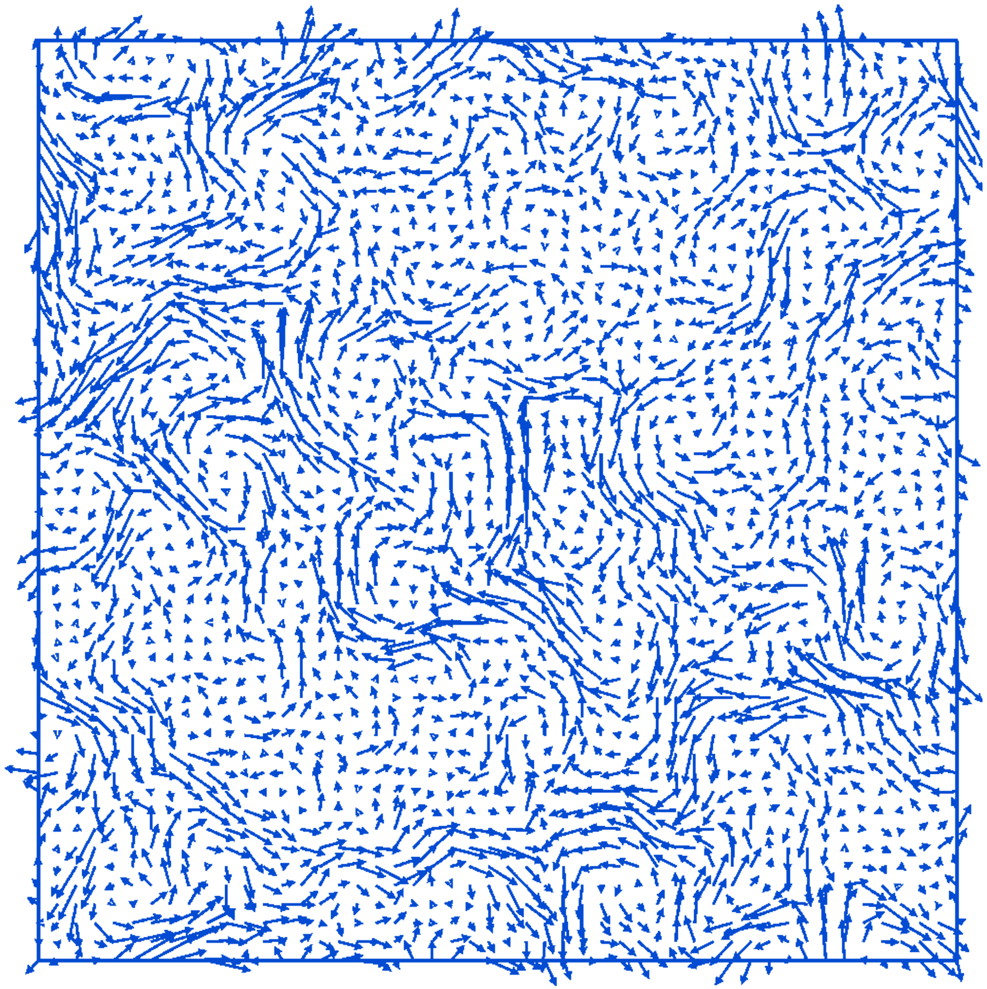
time →

cluster instability

Goldhirsch & Zanetti'93, McNamara'92

simulation by S. Luding (2005)
N=100,000; ε=0.9
periodic BC
homogeneously initialized

Event-driven MD: force-free granular gas

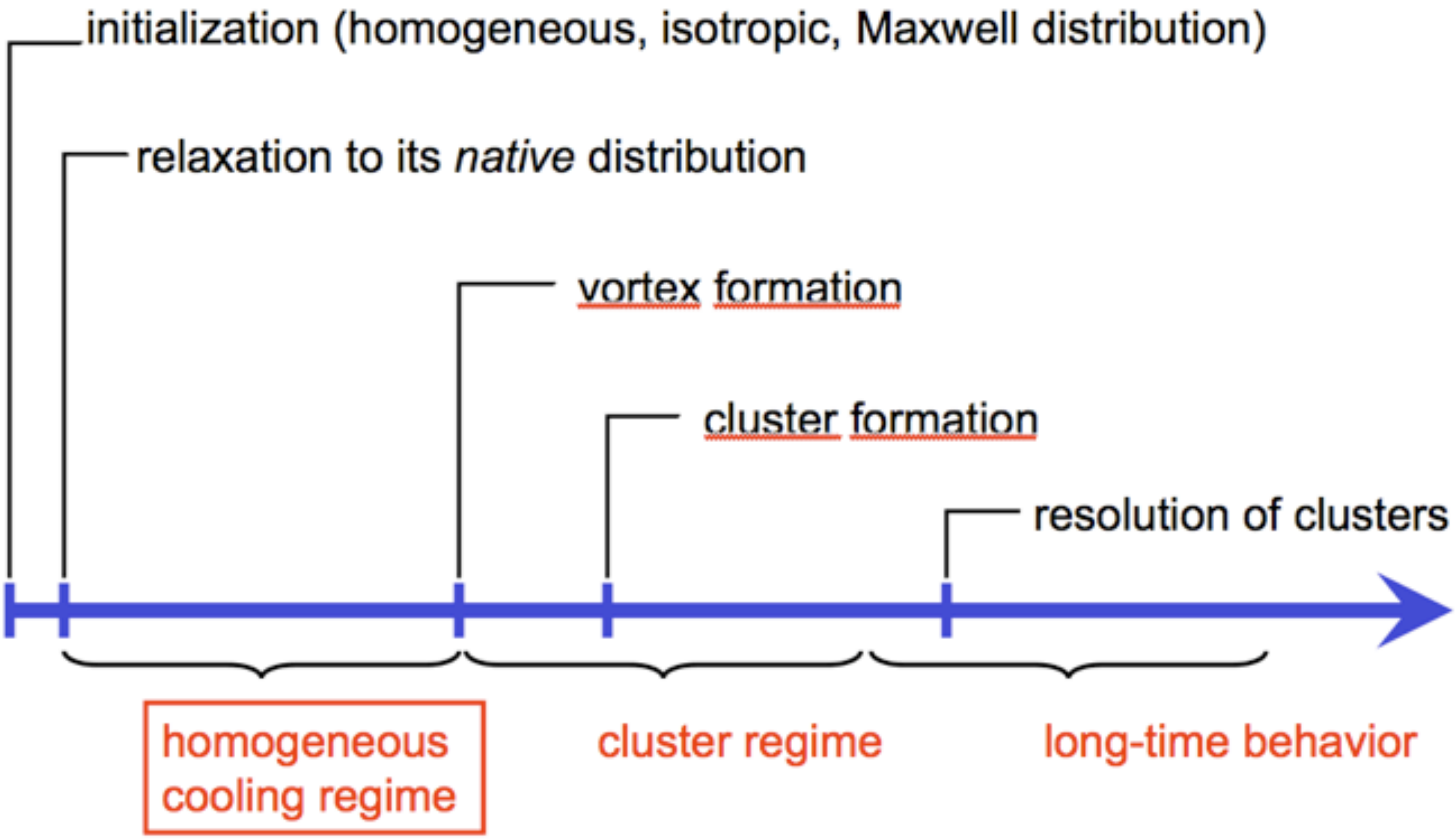


even before we see clusters, there is another instability

Brito, Ernst'98

simulation by TP (1998)
N=100,000; ε=0.9
periodic BC
homogeneously initialized

Event-driven MD: force-free granular gas



Yet more simple: consider homogeneous cooling state.

average energy loss per collision:

$$\Delta E = \frac{1}{2} m^{eff} \left(\overline{v}'^2_{12} - \overline{v}^2_{12} \right) = -\frac{1}{2} m^{eff} \overline{v}^2_{12} \left(1 - \varepsilon^2\right) \propto -\left(1 - \varepsilon^2\right) T$$

average number of collisions per unit time Δt:

$$\frac{\nu}{\Delta t} \propto n \sigma^2 \sqrt{T}$$

density

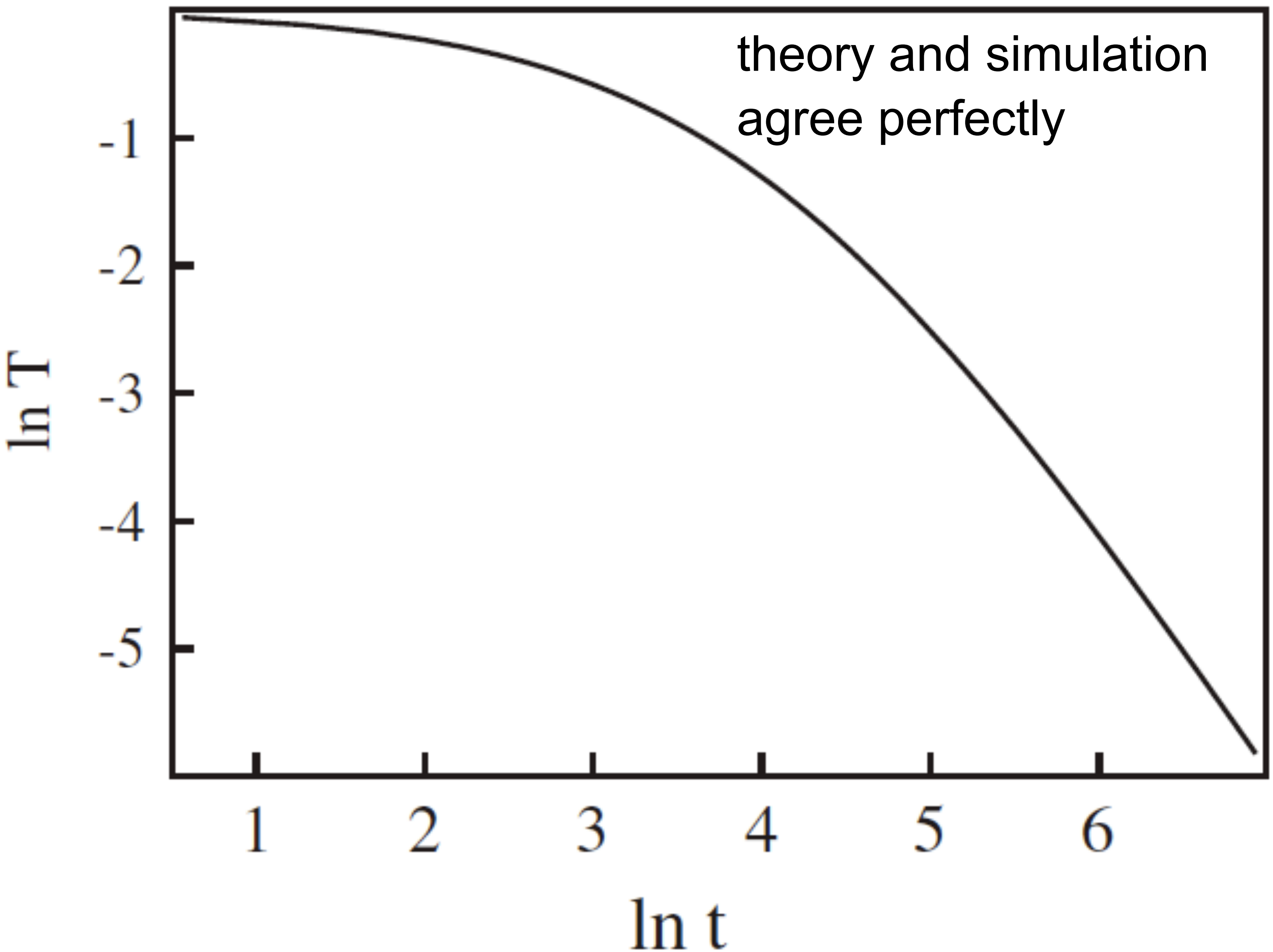
partice diameter

$$\frac{dT}{dt} \approx \frac{\Delta T}{\Delta t} \propto \frac{\nu \Delta E}{\Delta t} \propto -n\sigma^2 \left(1 - \varepsilon^2\right) T^{3/2}$$

$$T(t) = \frac{T_0}{\left(1 + t/\tau_0\right)^2} \qquad \tau_0^{-1} \propto n\sigma^2 \left(1 - \varepsilon^2\right) \sqrt{T_0}$$

Haff's law'83

insufficient since Maxwell distribution was assumed!


theory and simulation agree perfectly
-1
-2
-3
-4
-5
ln T
1
2
3
4
5
6
ln t

Velocity distribution function

- **Boltzmann-Enskog equation**

$$\frac{\partial}{\partial t} f(\vec{v}_1,t) + \vec{v} \cdot \nabla f(\vec{v}_1,t) = g_2(\sigma) I(f,f)$$

(the term $\vec{v} \cdot \nabla f(\vec{v}_1,t)$ is crossed out)

$$I(f,f) = \sigma^2 \int d\vec{v}_2 \int d\vec{e}\, \Theta(-\vec{v}_{12} \cdot \vec{e}) |\vec{v}_{12} \cdot \vec{e}| \times \left\{ \kappa f(\vec{v}_1'', t) f(\vec{v}_2'', t) - f(\vec{v}_1, t) f(\vec{v}_2, t) \right\}$$

$$g_2(\sigma) = \frac{2-\nu}{2(1-\nu)^3} \qquad \nu = \frac{\pi}{6} n \sigma^3$$

Carnahan, Starling'69

n particle number density
ν packing fraction
σ particle diameter

$$\kappa = \frac{D(\vec{v}_1'', \vec{v}_2'')}{D(\vec{v}_1, \vec{v}_2)} \frac{|g''|}{|g|}$$

- Jacobi determinant
- shortening of the collision cylinder

for ε=const: $\kappa = \frac{1}{\varepsilon^2}$

Velocity distribution function

- **Boltzmann-Enskog equation**

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for ε=const: $\kappa = \frac{1}{\varepsilon^2}$ for ε(g)-visco: $\kappa = 1 + \frac{11}{5} C_1 \delta' g^{1/5} + \frac{66}{25} C_1^2 \delta'^2 g^{2/5} + \ldots$

Velocity distribution function

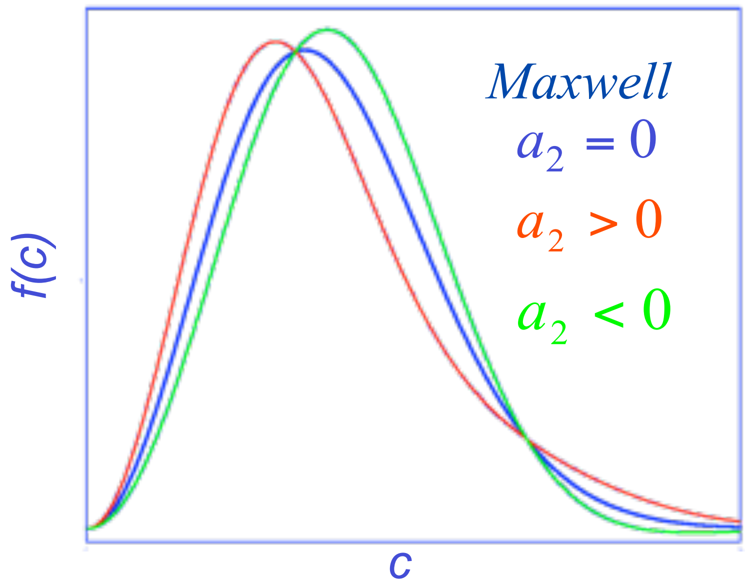
- **Sonine polynomials expansion**

$$\tilde{f}(c) = \frac{1}{\pi^{3/2}} e^{-c^2} \left\{ 1 + \sum_{k=1}^{\infty} a_k S_k\left(c^2\right) \right\}$$

$$S_0(x) = 1$$

$$S_1(x) = -x^2 + 3/2$$

$$S_2(x) = x^2/2 - 5x/2 + 15/8$$

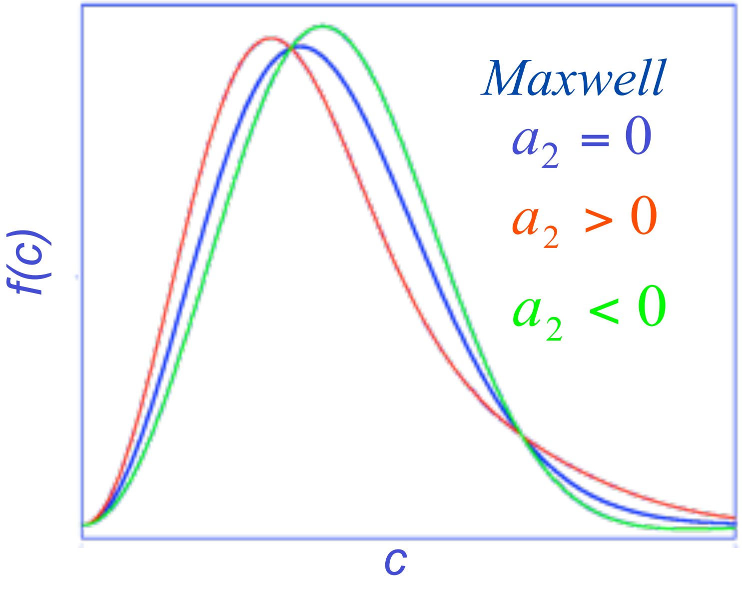


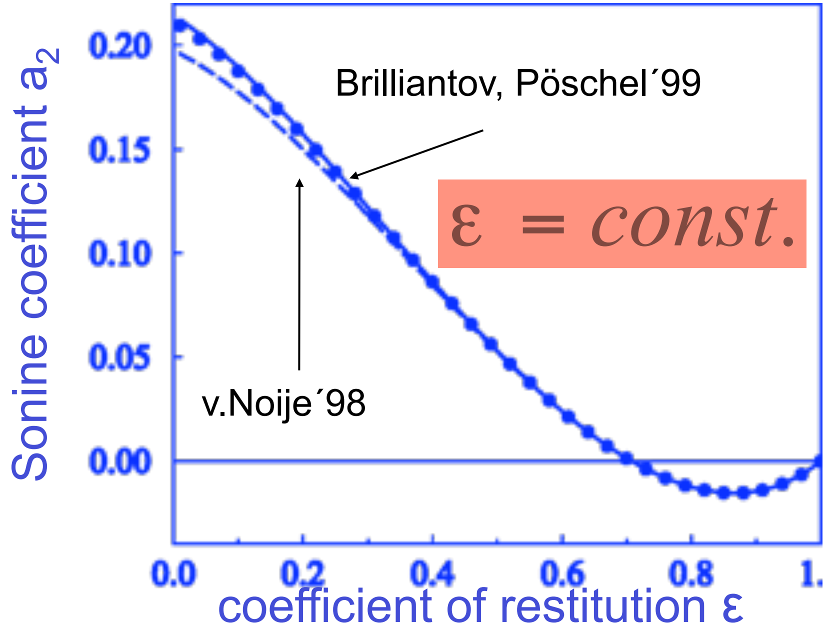
Velocity distribution function

- **Sonine polynomials expansion**

$$\tilde{f}(c) = \frac{1}{\pi^{3/2}} e^{-c^2} \left\{ 1 + \sum_{k=1}^{\infty} a_k S_k\left(c^2\right) \right\}$$

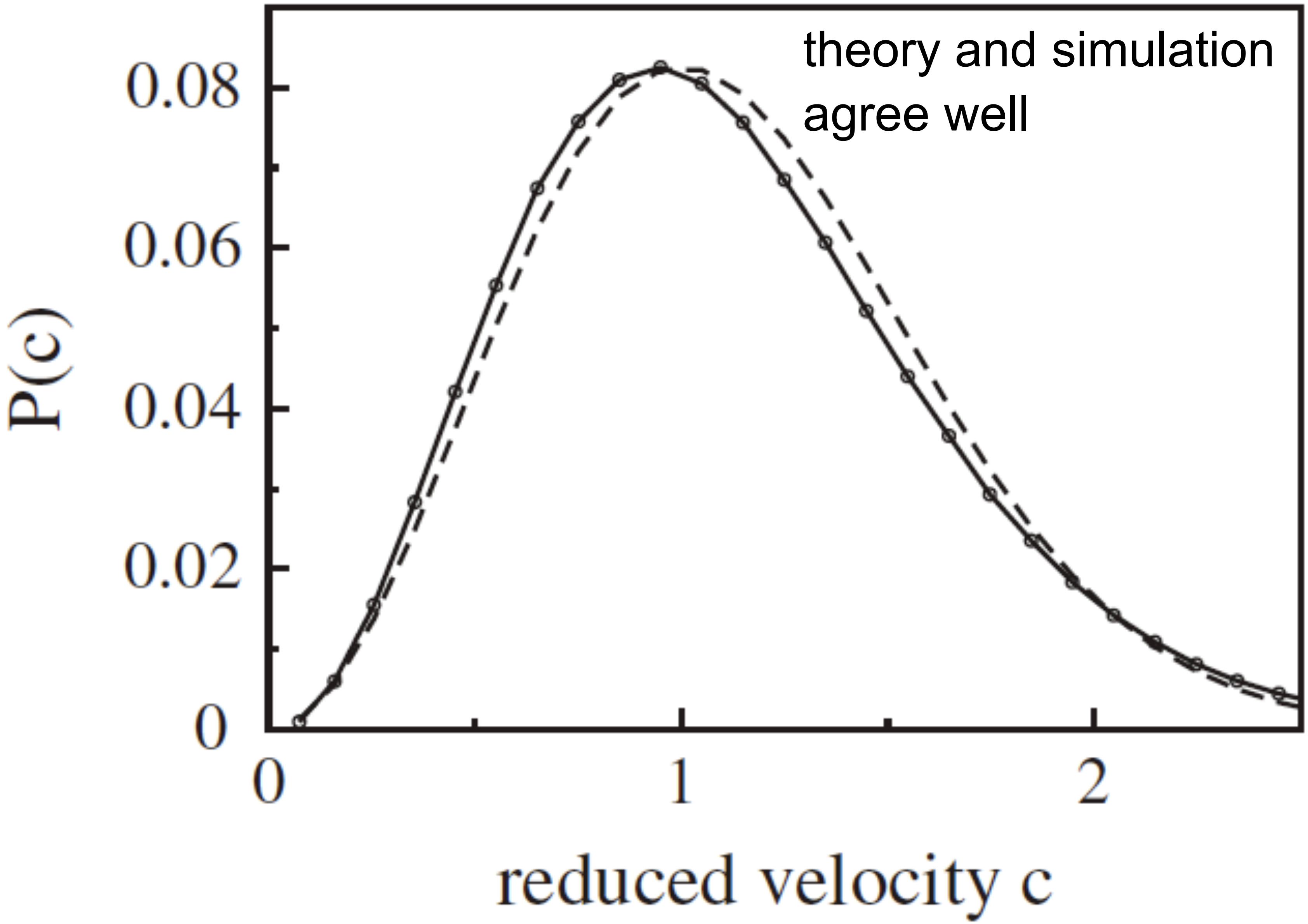
$$S_0(x) = 1$$
$$S_1(x) = -x^2 + 3/2$$
$$S_2(x) = x^2/2 - 5x/2 + 15/8$$





$$a_2 = \frac{16(1-\varepsilon)(1-2\varepsilon^2)}{81 - 17\varepsilon + 30\varepsilon^2(1-\varepsilon)} = const.$$

v.Noije, Ernst´98 Goldshtein, Shapiro´95

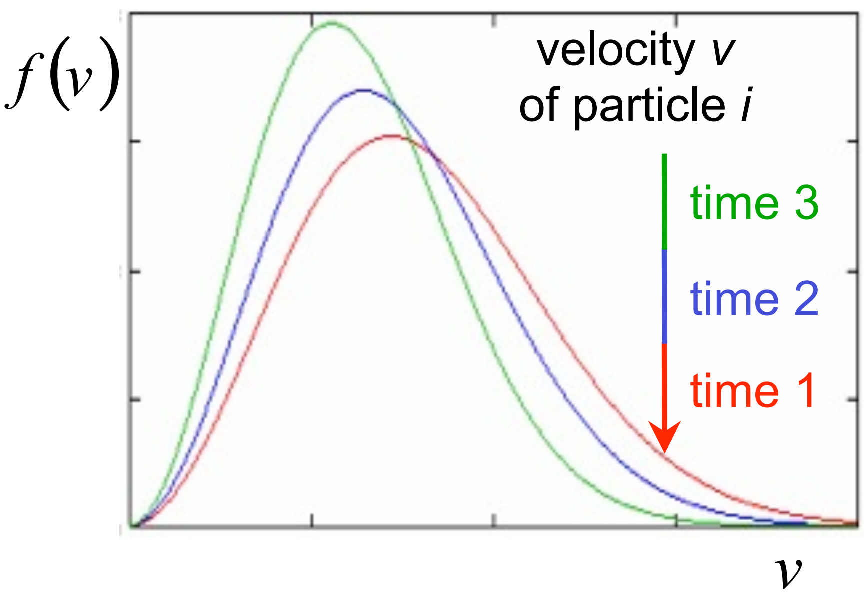

theory and simulation agree well
0.08
0.06
0.04
0.02
0
P(c)
0
1
2
reduced velocity c

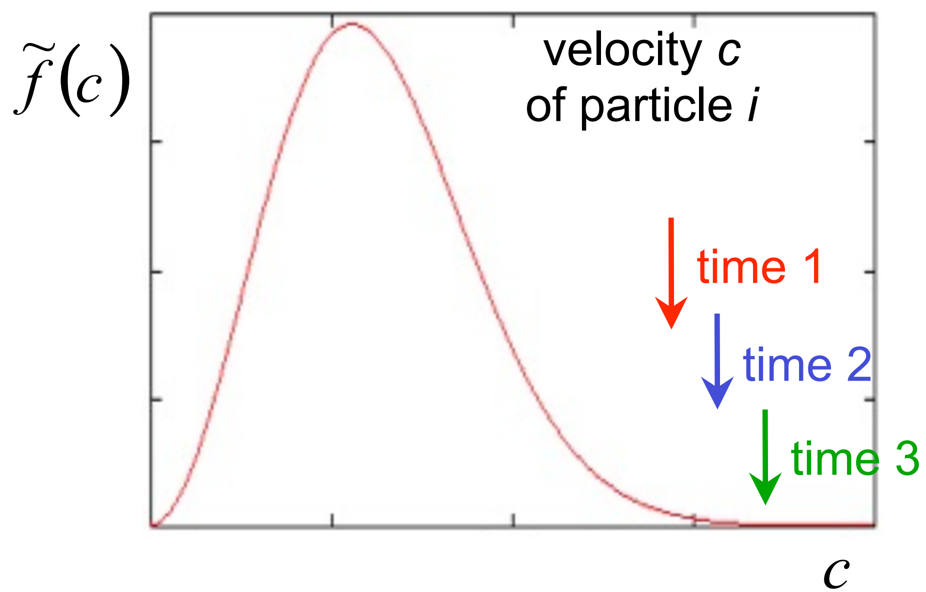
High-energy tail Esipov, Pöschel'97

Number of particles with reduced velocity is determined by the balance of 3 processes

$$c\,dc \equiv \frac{v}{v_T}\,d\,v$$

1. Loss due to collisions of particles from the interval *cdc*
2. Gain due to collisions of particles from other velocity intervals which enter the interval after a collision
3. Without collisions, due to the decay of the thermal velocity v_T





High-energy tail Esipov, Pöschel'97

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2. Gain due to collisions of particles from other velocity intervals which enter the interval after a collision
3. Without collisions, due to the decay of the thermal velocity v_T

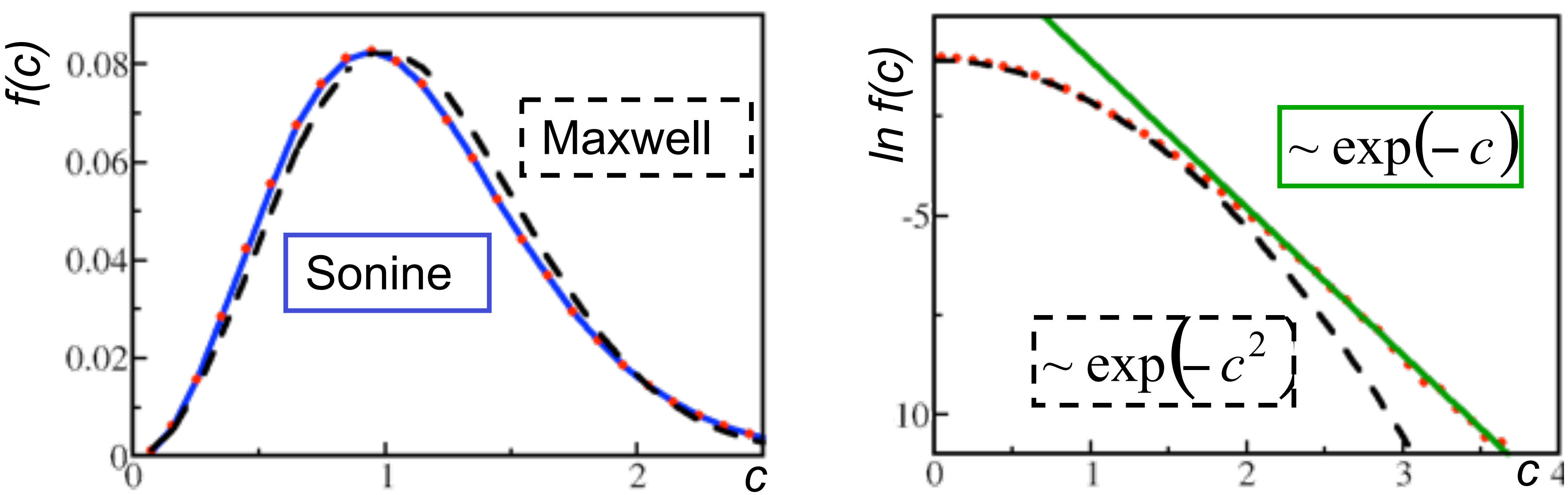
For dissipative gases process 2 may be neglected as compared with process 1 for the high-energy tail.

High-energy tail Esipov, Pöschel'97

Boltzmann equation – collision integral

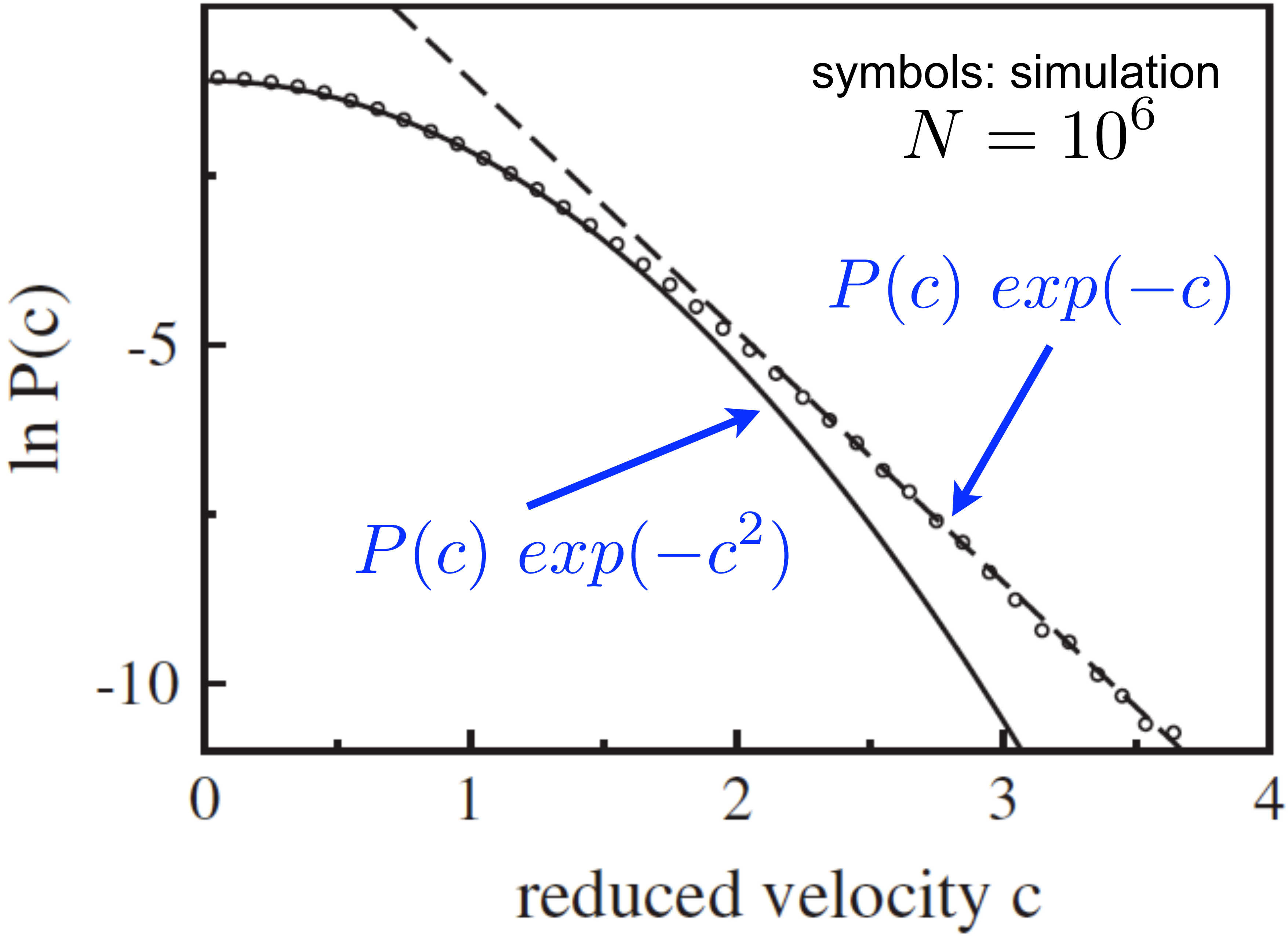
$$\tilde{I} = \int d\vec{c}_2 \int d\vec{e}\, \Theta\left(-\vec{c}_{12}\cdot\vec{e}\right)\left|\vec{c}_{12}\cdot\vec{e}\right| \left[\frac{1}{\varepsilon^2}\tilde{f}\left(\vec{c}_1^{\,\prime\prime},t\right)\tilde{f}\left(\vec{c}_2^{\,\prime\prime},t\right) - \tilde{f}\left(\vec{c}_1,t\right)\tilde{f}\left(\vec{c}_2,t\right)\right]$$

$$\approx -\pi\, c_1 \tilde{f}\left(\vec{c}_1\right) \quad \rightarrow \quad \tilde{f}(c) = const\, \exp\left(-\frac{3\pi}{\mu_2} c\right)$$



Direct Simulation Monte Carlo (DSMC) Bird'80

Schwager, Pöschel, Brilliantov'04


symbols: simulation
$N = 10^6$
$P(c)\ exp(-c)$
$P(c)\ exp(-c^2)$
-5
-10
0
1
2
3
4
ln P(c)
reduced velocity c

Boltzmann equation suggests a way for a much faster algorithm

Direct Simulation Monte Carlo

$$\frac{\partial}{\partial t} f(\vec{v}_1, t) + \vec{v} \times \nabla f(\vec{v}_1, t) = g_2(\sigma) I(f, f)$$

$$I(f,f) = \sigma^2 \int d\vec{v}_2 \int d\vec{e}\, \Theta(-\vec{v}_{12} \times \vec{e}) \, |\vec{v}_{12} \times \vec{e}| \quad \times$$

$$\left\{ \kappa\, f(\vec{v}_1'', t) f(\vec{v}_2'', t) - f(\vec{v}_1, t) f(\vec{v}_2, t) \right\}$$

Idea: BE is an equation for probabilities → fraction of particle having velocity v at time t

→ (at least) for equal particles, the prob. is proportional to the number of particles with v.

Assume a probability quantum = 1 particle. Then the probability quanta must behave in the same way as the particles themselves!

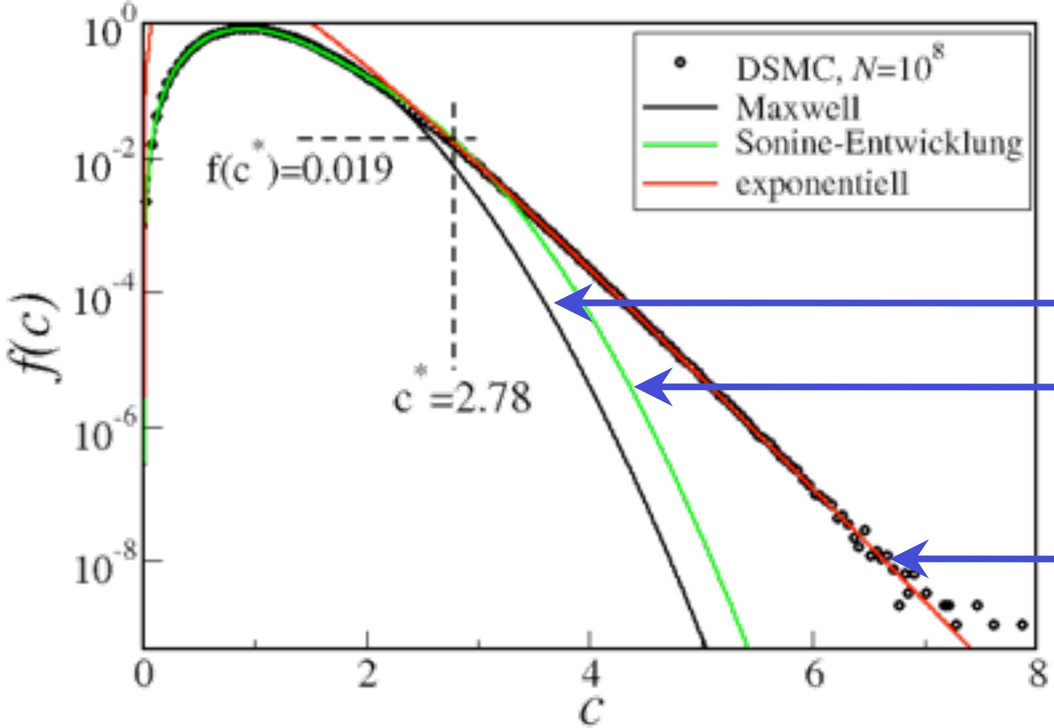
Boltzmann equation suggests a way for a much faster algorithm

Direct Simulation Monte Carlo

1. select 2 particles at random (their chance to select a particle from the interval v is equal to their occurance)
2. select a random unit vector (if isotropy holds, all directions of collisions are equal)
3. the pair is accepted if

$$|\vec{e}\cdot\vec{v}_{12}| = |\vec{e}\cdot(\vec{v}_1 - \vec{v}_2)| > \mathtt{rand}[0,1)\, v_{12}^{\max}$$

4. apply the collision rule to compute the new velocities
5. continue with step 1



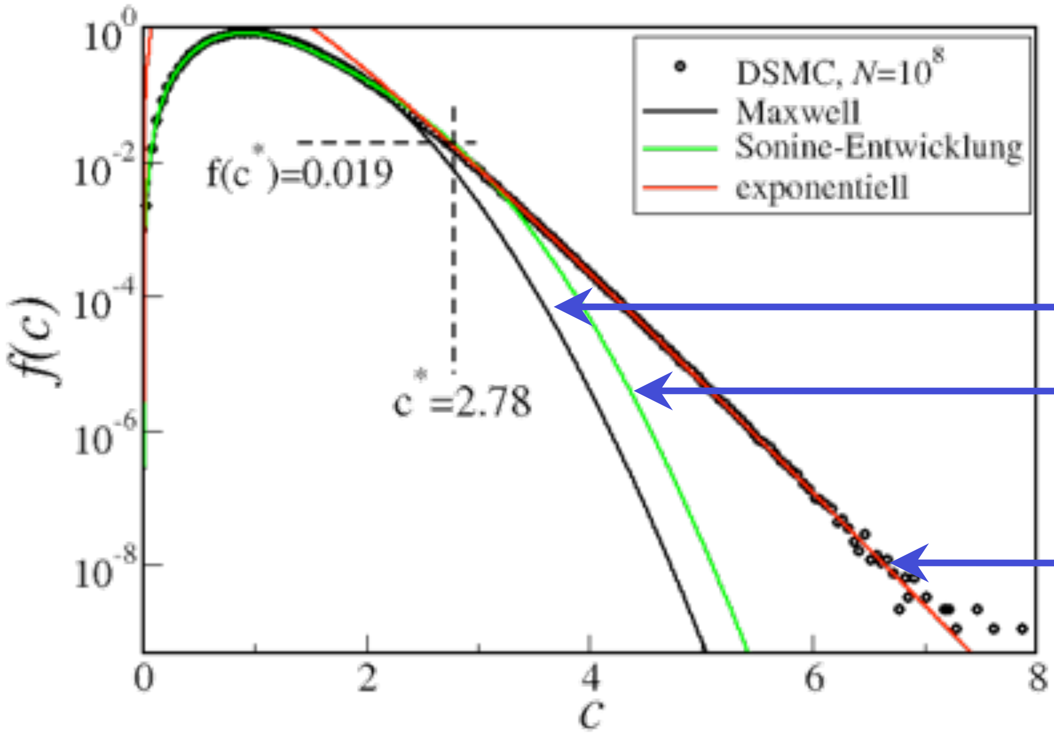
$\varepsilon = 0.3 \qquad N = 10^8$

Maxwell

Sonine expansion
Goldshtein, Shapiro´95,
v.Noije, Ernst´98

exponential tail
Esipov, Pöschel'96

Pöschel, Brilliantov'06



$\varepsilon = 0.3$

$N = 10^8$

Maxwell

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Goldshtein, Shapiro´95,
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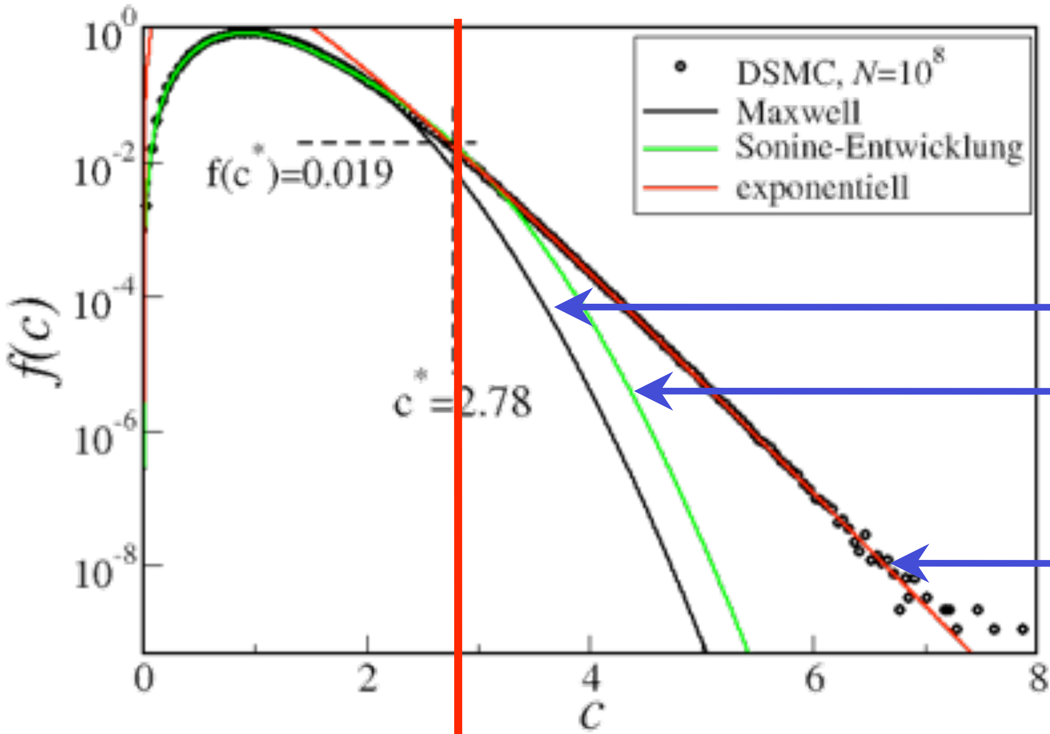

$\varepsilon = 0.3$

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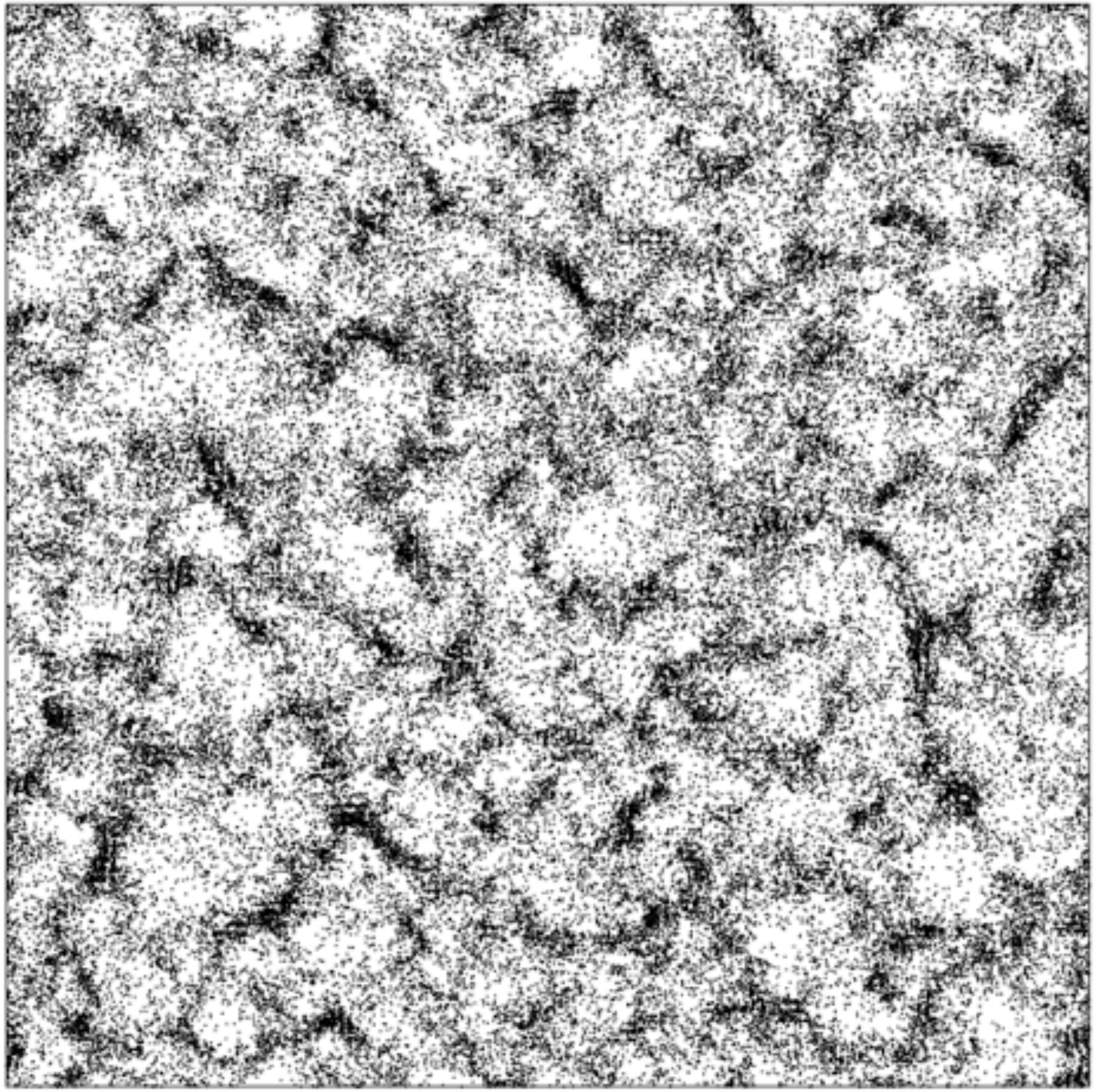
Maxwell

Sonine expansion
Goldshtein, Shapiro´95,
v.Noije, Ernst´98

exponential tail
Esipov, Pöschel'96

Sonine expansion | overpopulated tail

Pöschel, Brilliantov'06



$$N = 10^8$$

Are there correlations (structures) in homogeneous granular gases?

fundamental (but trivial) answer:

If $v_x \; v_y \; v_z$ components of the velocity are independent

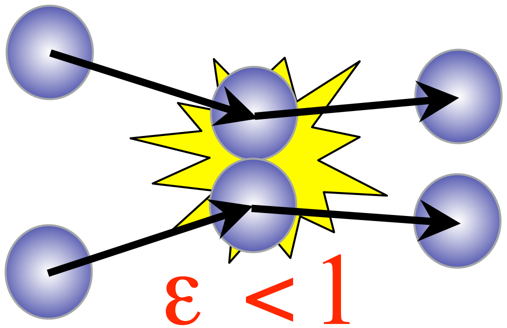
$$F(v) = f(|v|) \neq f(v_x)$$ isotropic

→ $$f(v_x) = a\, e^{-b v_x^2}$$ Maxwell distribution

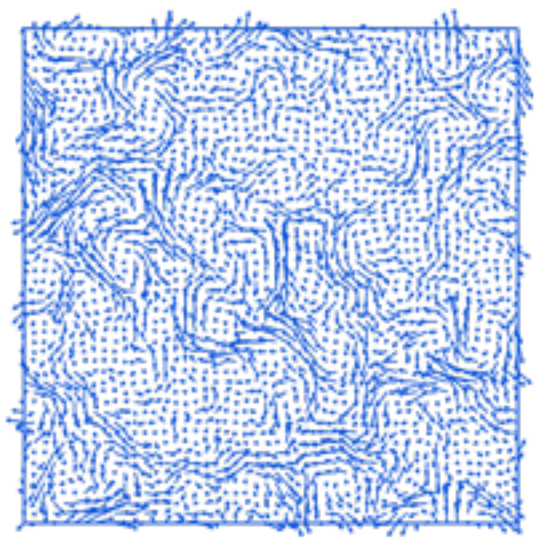
There are deviations from the Maxwell-distribution
→ There are correlations

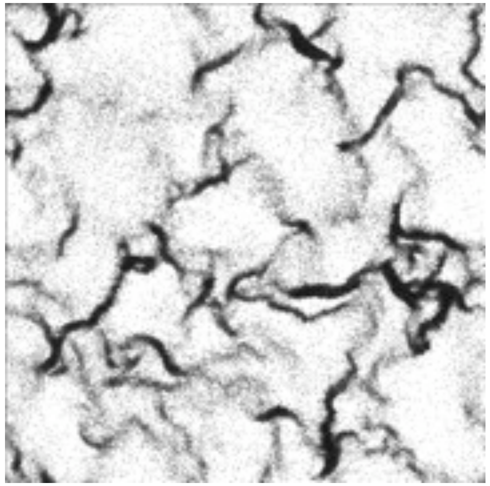
Are there correlations (structures) in homogeneous granular gases?

Bar-Lev
Brey
Brilliantov
Brito
Ernst
Goldhirsch
Luding
Mareschal
M. B. Marconi
McNamara
Noskowicz
Piasecki
Pöschel
Puglisi
Soto
Zippelius



precollisional velocity correlations





Are there correlations (structures) in homogeneous granular gases?

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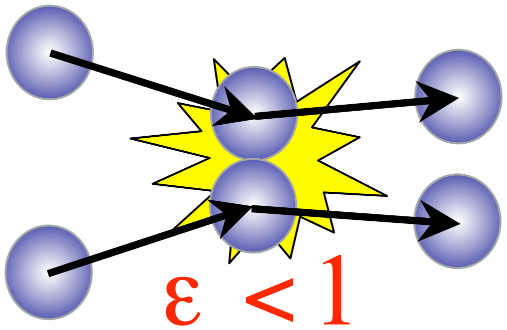
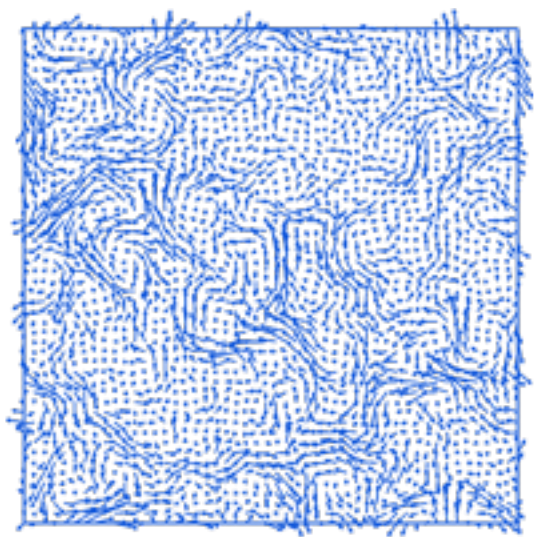
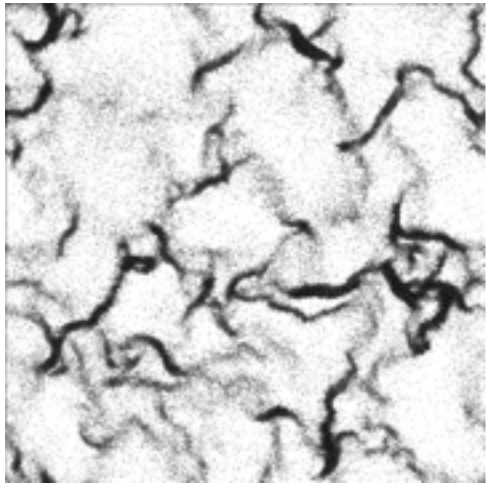


precollisional velocity correlations





multi-particle correlations

Rotating Particles

collision law

$$\vec{v}_1' = \vec{v}_1 - \frac{1+\varepsilon^n}{2}\vec{v}_{ij}^{\,n} + \frac{\tilde{J}\left(\varepsilon^t - 1\right)}{2\left(\varepsilon^t + 1\right)}\vec{v}_{ij}^{\,t}$$

$$\vec{v}_2' = \vec{v}_2 + \frac{1+\varepsilon^n}{2}\vec{v}_{ij}^{\,n} - \frac{\tilde{J}\left(\varepsilon^t - 1\right)}{2\left(\varepsilon^t + 1\right)}\vec{v}_{ij}^{\,t}$$

$$\vec{\omega}_1' = \vec{\omega}_1 - \frac{\varepsilon^t - 1}{2\sigma\left(\tilde{J}+1\right)}\left(\vec{e}\times\vec{v}_{ij}^{\,t}\right)$$

$$\vec{\omega}_2' = \vec{\omega}_2 - \frac{\varepsilon^t - 1}{2\sigma\left(\tilde{J}+1\right)}\left(\vec{e}\times\vec{v}_{ij}^{\,t}\right)$$

$$\tilde{J} \equiv J/\left(m\sigma^2\right)$$

Rotating Particles

collision law

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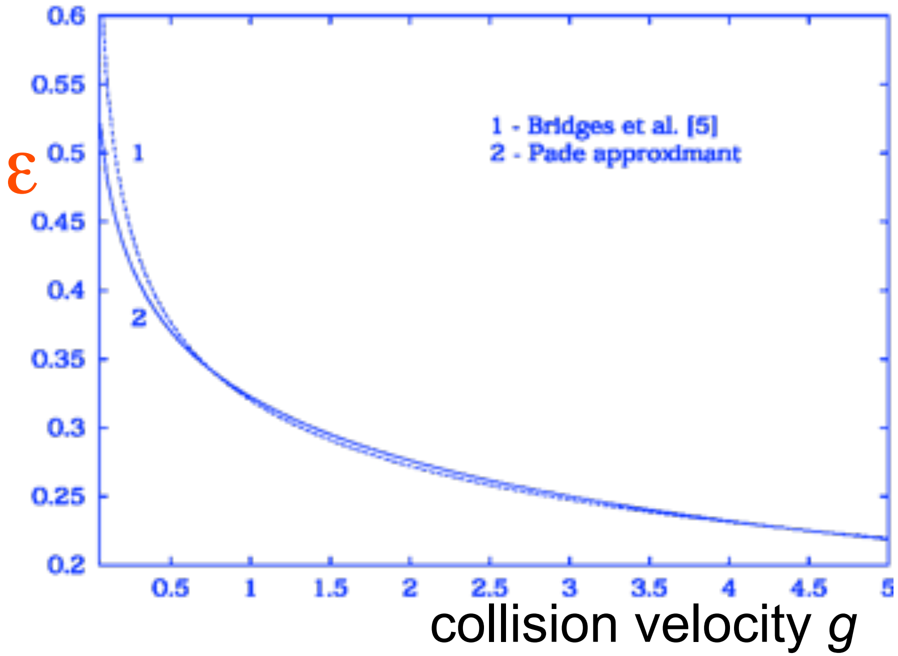
$$\tilde{J} \equiv J/\left(m\sigma^2\right)$$

remark: for realistic materials

$$\varepsilon_n = \varepsilon_n\left(g_n\right)$$



Schwager, Pöschel'01

Rotating Particles

collision law

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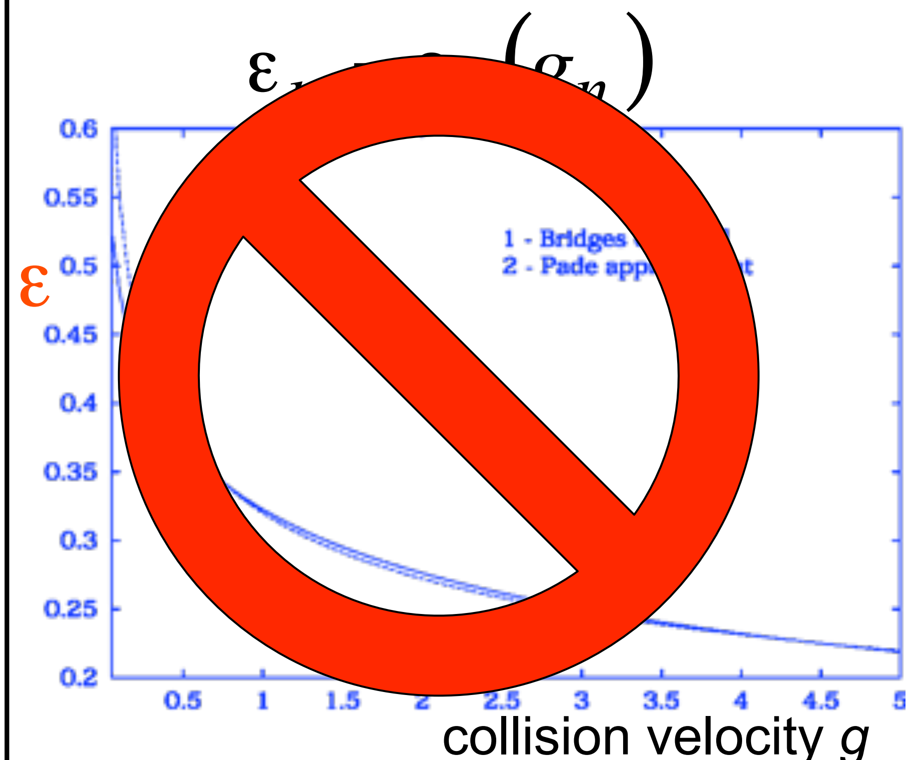

Schwager, Pöschel'01

here we assume $\varepsilon_n = const.$

Rotating Particles

collision law

$$\vec{v}_1' = \vec{v}_1 - \frac{1+\varepsilon^n}{2}\vec{v}_{ij}^{\,n} + \frac{\tilde{J}\left(\varepsilon^t - 1\right)}{2\left(\varepsilon^t + 1\right)}\vec{v}_{ij}^{\,t}$$

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Rotating Particles

collision law

$$\vec{v}_1' = \vec{v}_1 - \frac{1+\varepsilon^n}{2}\vec{v}_{ij}^n + \frac{\tilde{J}(\varepsilon^t - 1)}{2(\varepsilon^t + 1)}\vec{v}_{ij}^t$$

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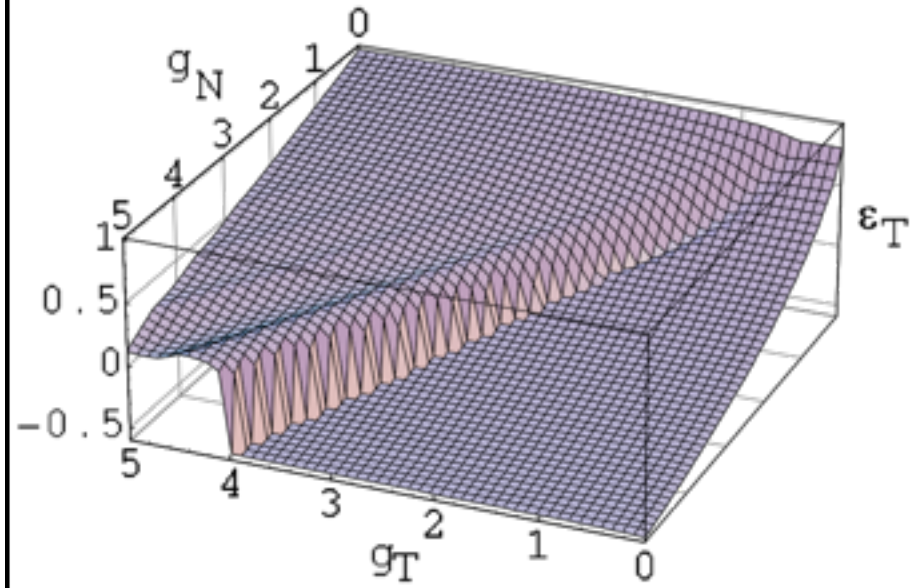

Brilliantov, Spahn, Hertzsch, Pöschel'96

Rotating Particles

collision law

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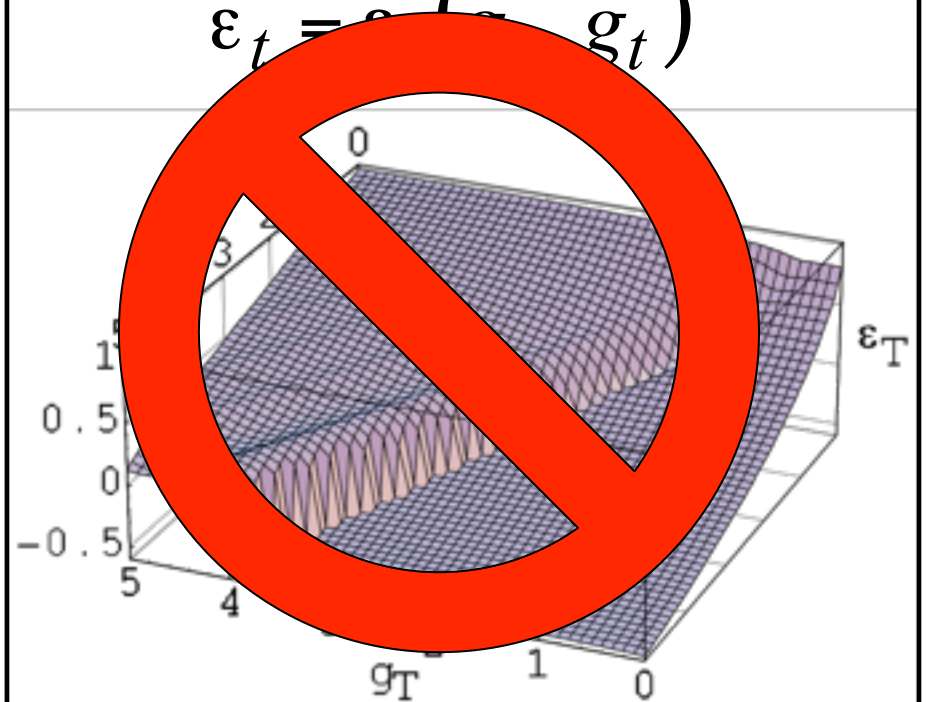

Brilliantov, Spahn, Hertzsch, Pöschel'96

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Rotating Particles

collision law

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$$\tilde{J} \equiv J/(m\sigma^2)$$

technically complicated distribution function $f(\vec{\omega})$

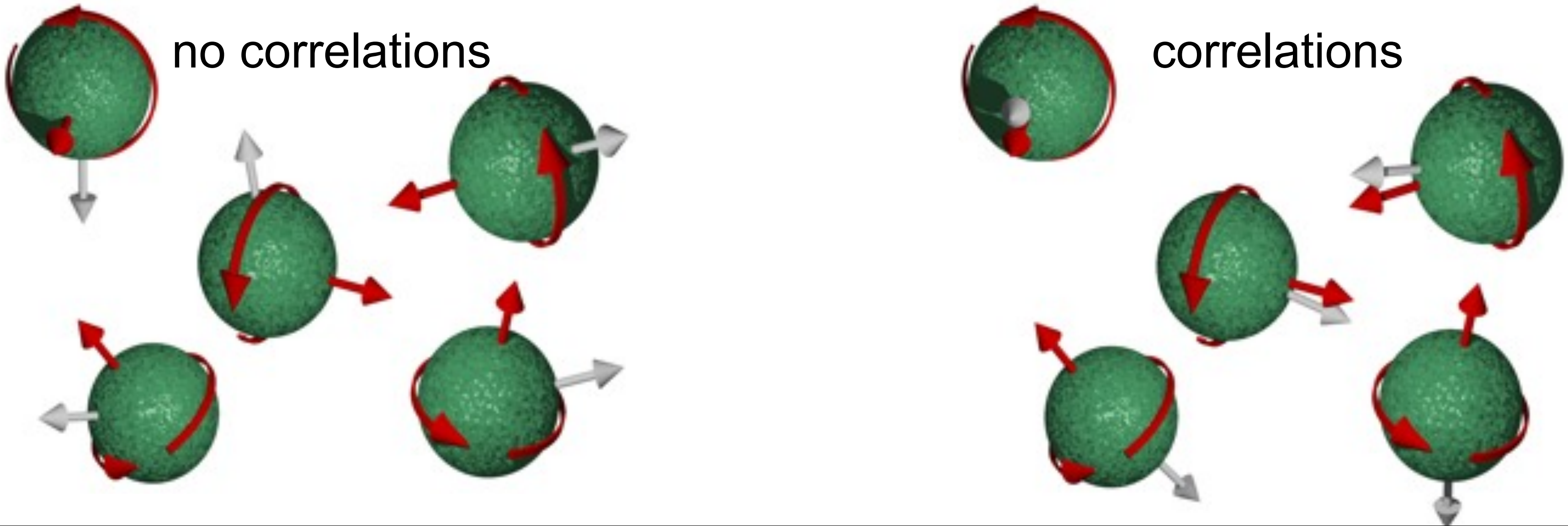
⬇

very similar effects as for $f(\vec{v})$, but more pronounced

Goldhirsch, Bar-Lev, Noskovich'05

New effect: correlation between rotation and translation

consider the directions of translation and rotation



these correlations are characterized by $C = \left\langle \frac{(\vec{v} \times \vec{\omega})^2}{v^2\, \omega^2} \right\rangle = \langle \cos^2 \Theta \rangle$

New effect: correlation between rotation and translation

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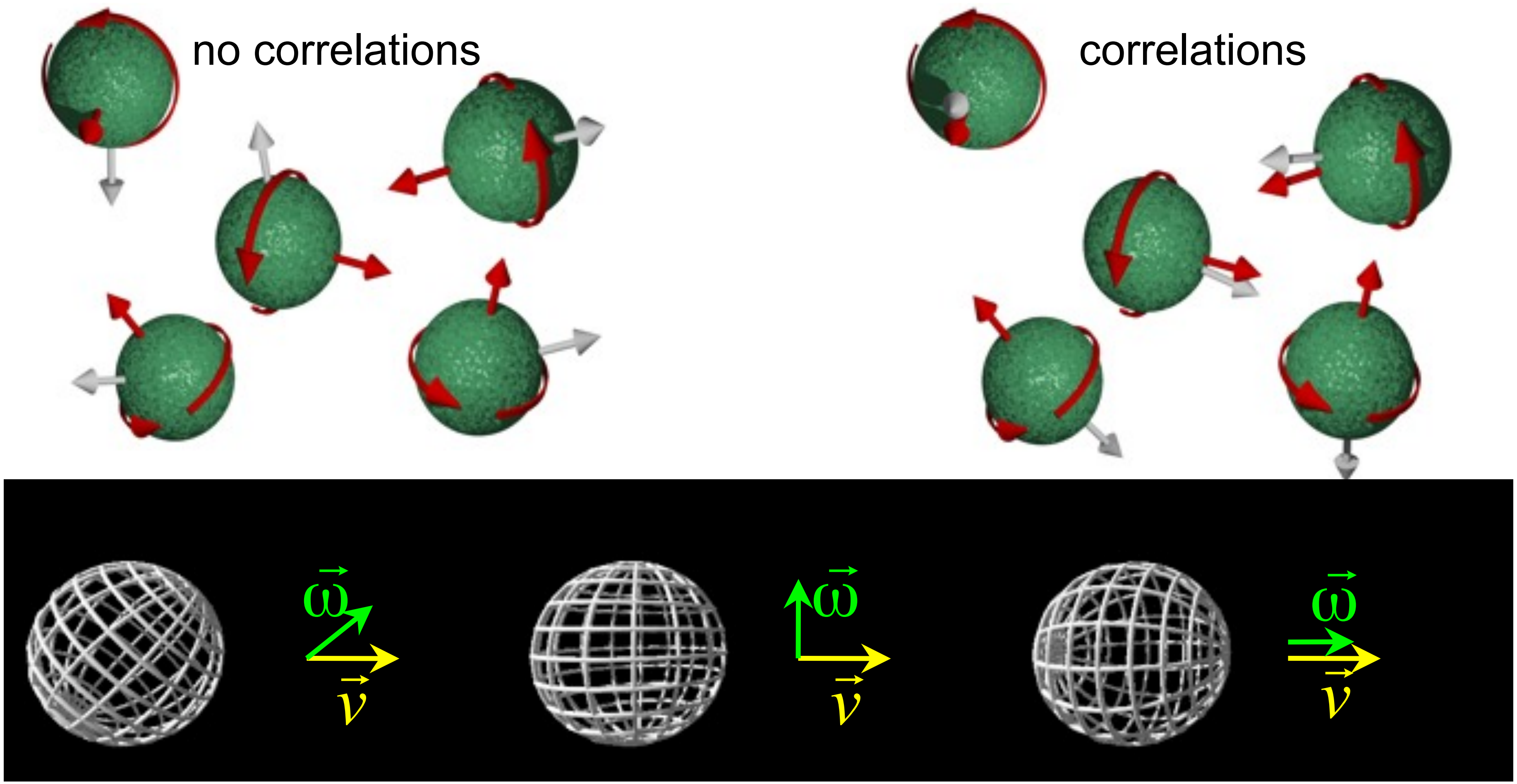

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Correlation between rotation and translation

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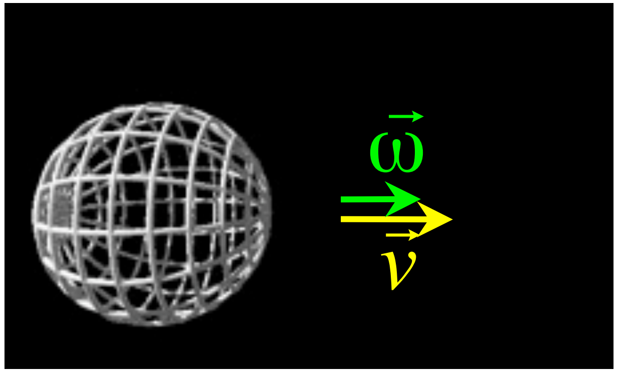
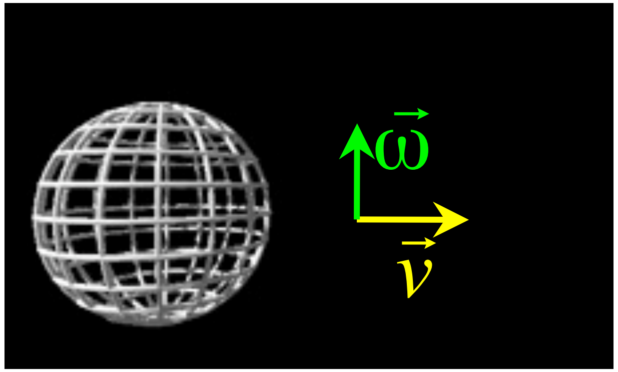


rifled cannon ball



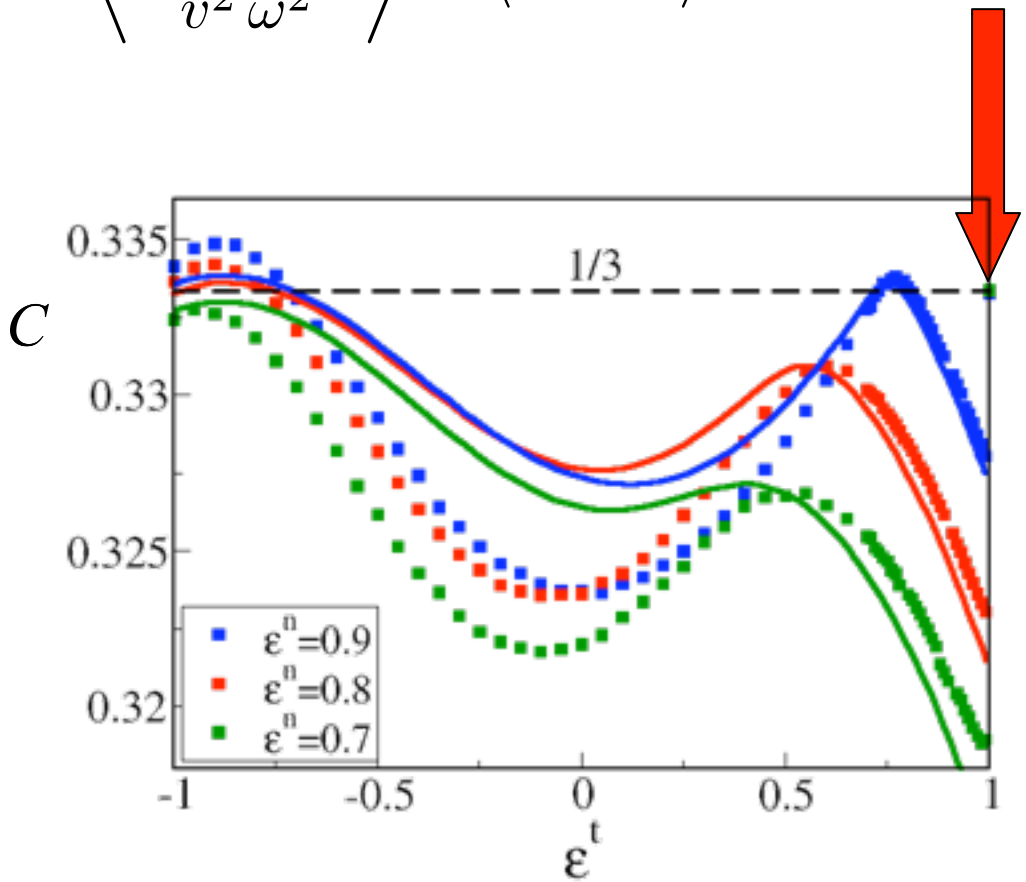
sliced tennis ball

$$C = \left\langle \frac{(\vec{v} \times \vec{\omega})^2}{v^2\,\omega^2} \right\rangle = \langle \cos^2 \Theta \rangle$$

Correlation between rotation and translation

$$C = \left\langle \frac{(\vec{v} \times \vec{\omega})^2}{v^2\,\omega^2} \right\rangle = \langle \cos^2 \Theta \rangle$$

no correlations (molecular gas): C=1/3



symbols: DSMC Simulationen, $N=10^7$

lines: analytical theory

Brilliantov, Pöschel, Kranz, Zippelius PRL‘07,09

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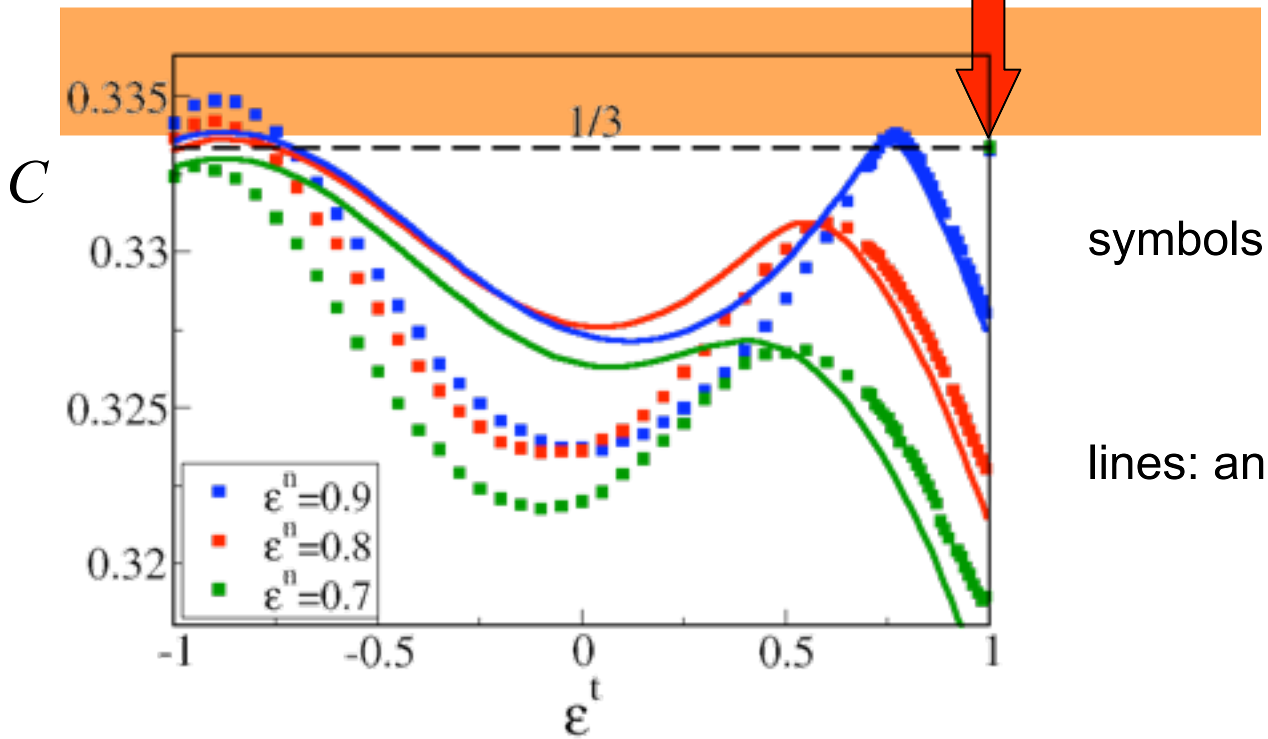

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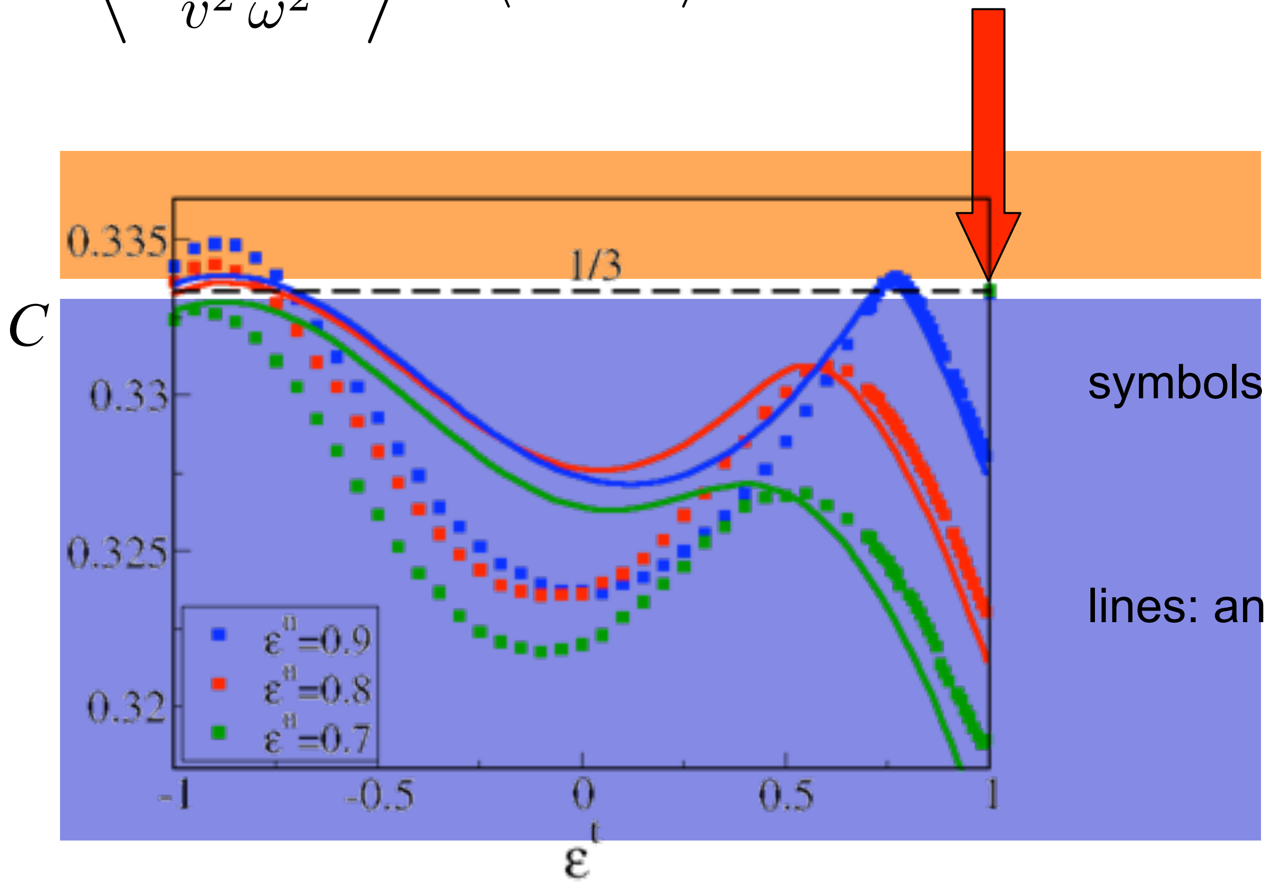

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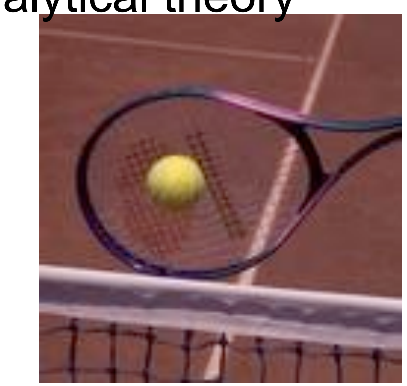


Brilliantov, Pöschel, Kranz, Zippelius PRL‘07,09

Correlation between rotation and translation

$$\left\langle \cos^2\Theta \right\rangle_\infty - \frac{1}{3} = -\frac{5}{6}\,\frac{A^{(0)} + (A-C)B^{(0)}/B + \left(B^{(0)} + C^{(0)}\right)r^*}{A^{(1)} - 40C + (A-C)B^{(1)}/B + \left(40B + B^{(1)}\right)r^*}$$

$$r^* = \left.\frac{T_{rot}}{T_{tr}}\right|_\infty$$

Huthmann, Zippelius'97

$$\eta_n = \frac{1}{2}(1+\varepsilon_n) \qquad \eta_t = \frac{q}{2}\left(\frac{1-\varepsilon_t}{1+q}\right) \qquad q = \frac{I}{mR^2}$$

$$A^{(0)} = \frac{16}{3}\frac{\eta_t^3}{q}\left(\frac{2\eta_t}{q} - 1\right) - \frac{2}{3}\frac{\eta_t^2}{q}\left(\frac{8\eta_t}{q} - 3\right) + \frac{1}{3}\frac{\eta_t}{q}\left(\frac{\eta_t}{q} - 1\right) + \frac{8}{3}\frac{\eta_t}{q}\left(\frac{\eta_t}{q} - 1\right)\eta_n(\eta_n - 1)$$

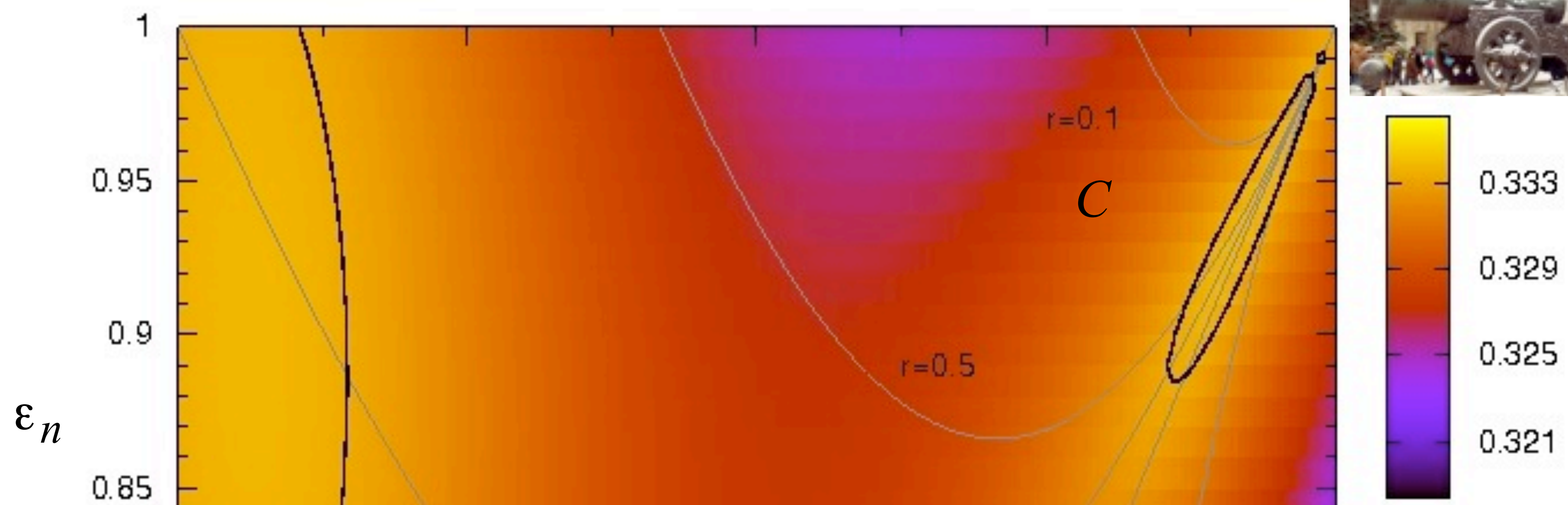
$$A^{(1)} = \frac{4\eta_t\eta_n^2}{q}\left(\frac{\eta_t}{q} - 1\right) + \frac{1}{3}\frac{\eta_t^2}{q}\left(\frac{24\eta_t}{q} - 37\right) + \frac{5}{6}\frac{\eta_t}{q}\left(\frac{9\eta_t}{q} - 29\right) - \frac{8\eta_t^3}{q}\left(\frac{2\eta_t}{q} - 1\right)$$
$$+ \frac{4}{3}\frac{\eta_t\eta_n}{q}\left(\frac{3\eta_t}{q} - 14\right) - 12\eta_t\eta_n + 22(\eta_t + \eta_n) - 6\left(\eta_t^2 + \eta_n^2\right)$$

$$B^{(0)} = \frac{1}{3}\frac{\eta_t^2}{q}\left(\frac{16\eta_t}{q}\left(\frac{\eta_t}{q} - 1\right) + 5\right) \qquad B^{(1)} = -\frac{2}{3}\frac{\eta_t^2}{q}\left(\frac{8\eta_t}{q}\left(\frac{\eta_t}{q} - 1\right) + 1\right)$$

$$B^{(1)} = \frac{2}{3}\frac{\eta_t^2}{q}\left(8\eta_t(\eta_t - 1) + 4\eta_n(\eta_n - 1) + 3\right)$$

Brilliantov, Pöschel, Kranz, Zippelius PRL'07,09

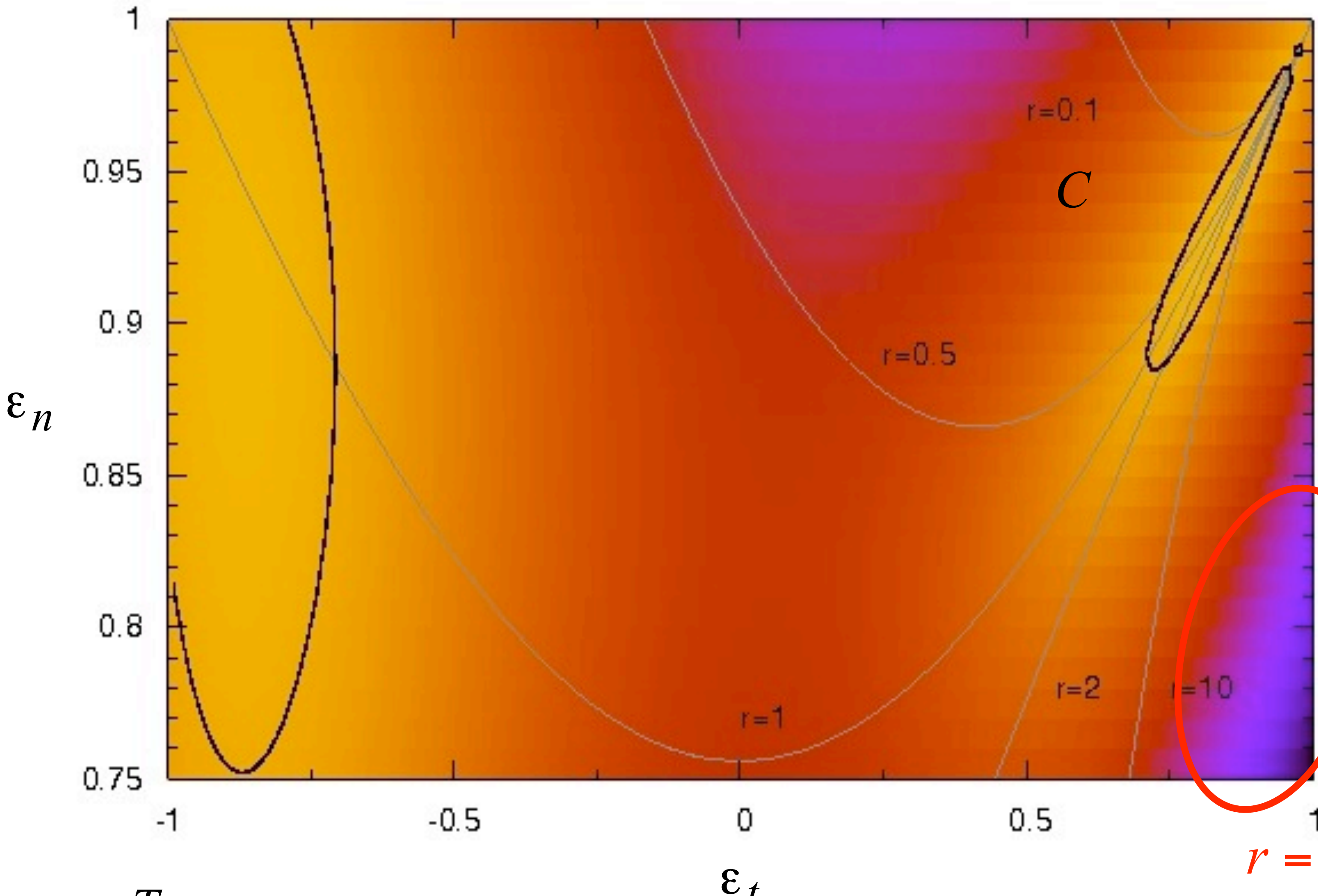
Correlation between rotation and translation



$$r = \frac{T_{rot}}{T_{tr}}$$

Brilliantov, Pöschel, Kranz, Zippelius PRL'07,09

Correlation between rotation and translation



$$r = \frac{T_{rot}}{T_{tr}} >> 1$$

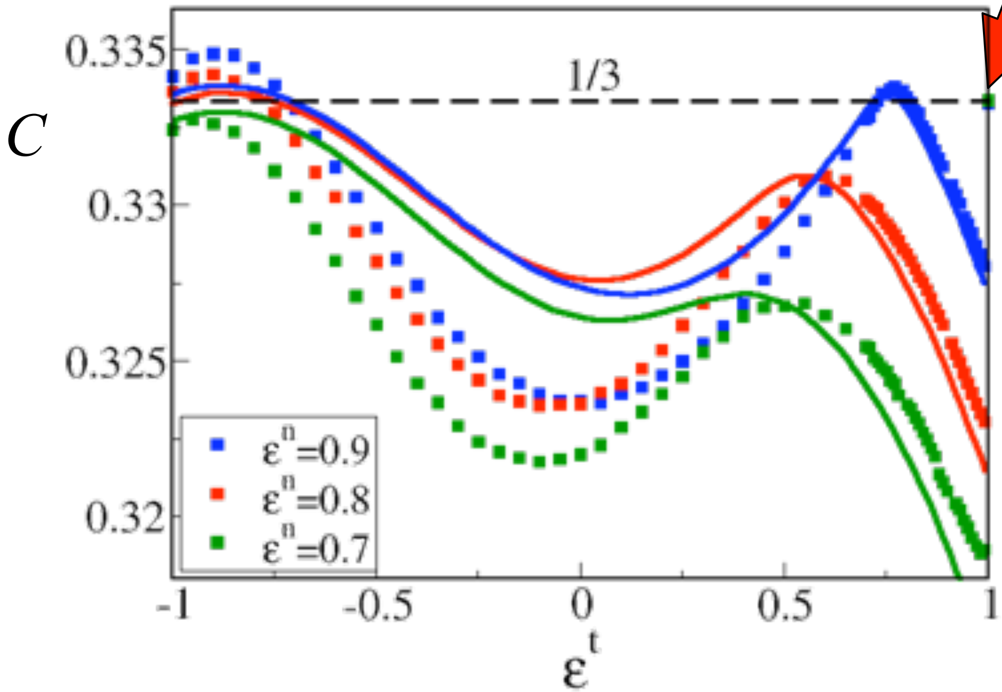
$$r = \frac{T_{rot}}{T_{tr}}$$

Brilliantov, Pöschel, Kranz, Zippelius PRL‘07,09

Correlation between rotation and translation

$$C = \left\langle \frac{(\vec{v} \times \vec{\omega})^2}{v^2\, \omega^2} \right\rangle = \langle \cos^2 \Theta \rangle$$

no correlations (molecular gas): C=1/3



$$\lim_{\varepsilon_t \to 1} C(\varepsilon_n, \varepsilon_t) = \frac{1}{3} - \frac{3}{8} \frac{(1-\varepsilon_n)}{(7-\varepsilon_n)} \neq \frac{1}{3}$$

no smooth transition
to the elastic case !!!

Brilliantov, Pöschel, Kranz, Zippelius PRL‘07,09

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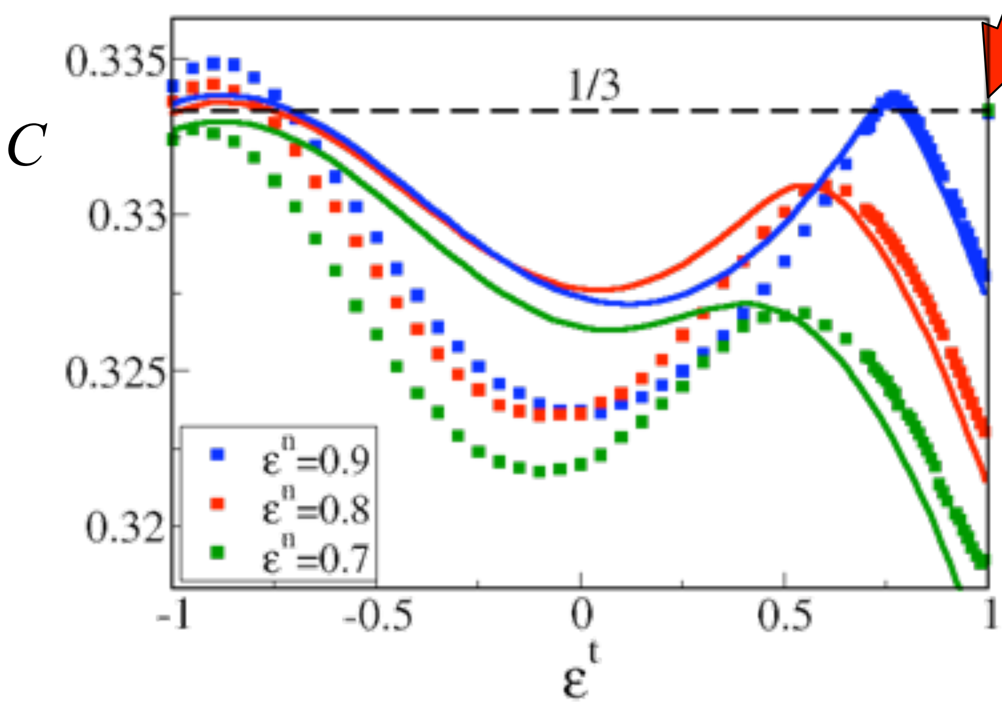

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no smooth transition
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Be careful when expanding around the elastic limit !

Brilliantov, Pöschel, Kranz, Zippelius PRL'07,09

Correlation between rotation and translation

consider the relaxation time

- **infinite relaxation time in the elastic limit:**

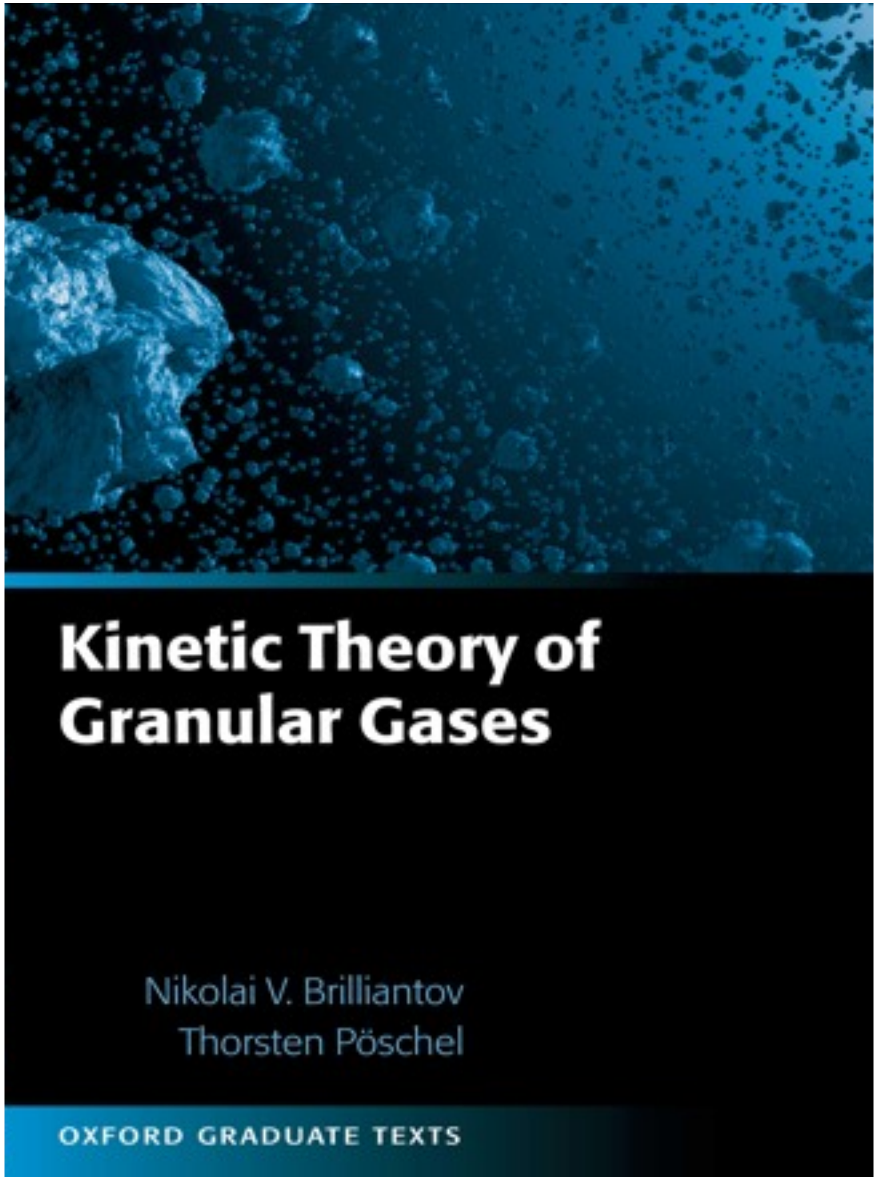
  (finite asymptotic deviation from C=1/3 at vanishing trend)

→ Is the concept of a granular gas problematic???

There are many more structures

- shockwaves
- supersonic phenomena (also found in experiments - Swinney-group)
- aging
- resolution of clusters after *very* long time

etc.



More on Kinetic Theory of Granular Gases

- Mechanics of particle collisions
- Velocity distribution function
- One-particle transport. Self-diffusion and Brownian motion
- Transport processes and kinetic coefficients
- Structure formation
- Numerical methods

Oxford University Press (2004)

Structures in Granular Gases

Computer Experiments meet Kinetic Theory

thanks to

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