Locomotion & Nonlinear Dynamics:
climbing cockroaches, shifting sand & running robots

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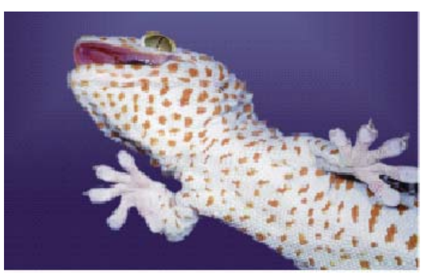
FUNDING NSF, Army Research Lab, Burroughs Wellcome Fund

Locomotion is a hallmark of biological systems

Dickinson et al, *Science* 2000

Animals go anywhere



TERRESTRIAL BIOLOGICAL LOCOMOTION

Ion channels to muscle physiology to musclo-skeletal dynamics to substrate interaction to ecosystems

Many DOF, hierarchical organization, nonlinear

Slowed 10x

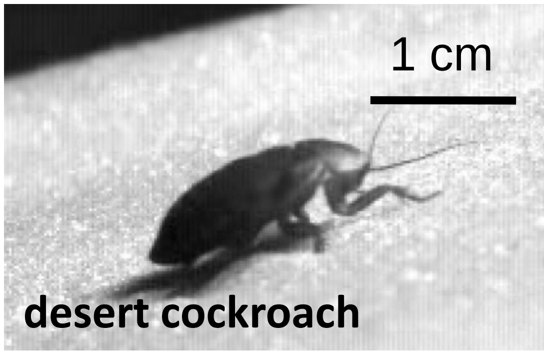
The terrestrial environment is mechanically complex:

Organisms interact with materials to which they adhere, which can yield, flow and fracture

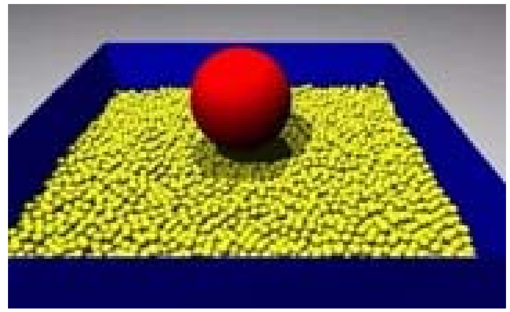
With Tom Libby

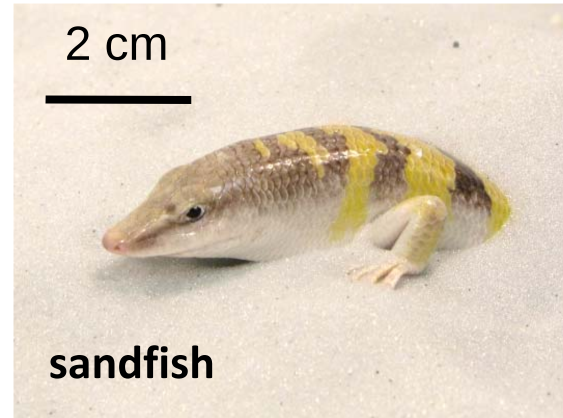
To discover principles of locomotion requires control of substrate

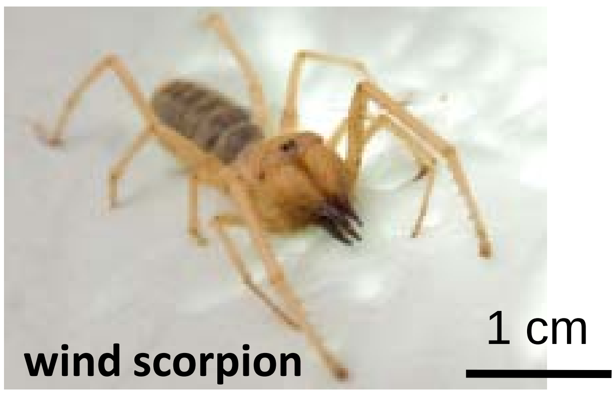
Complex Rheology And Biomechanics Lab

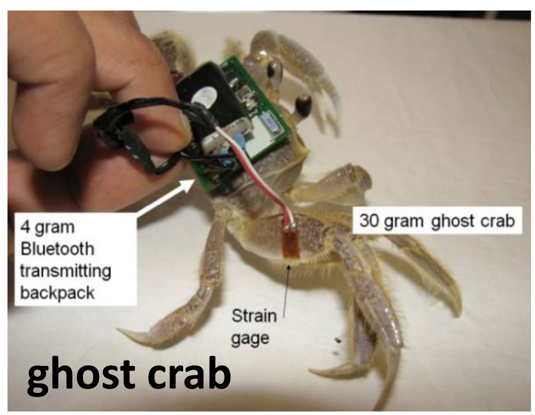


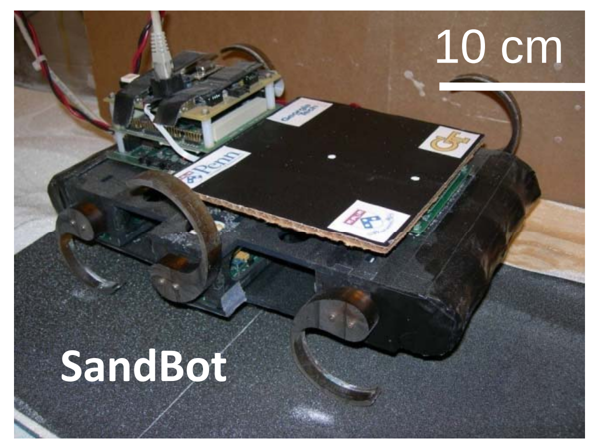
Penetration & drag

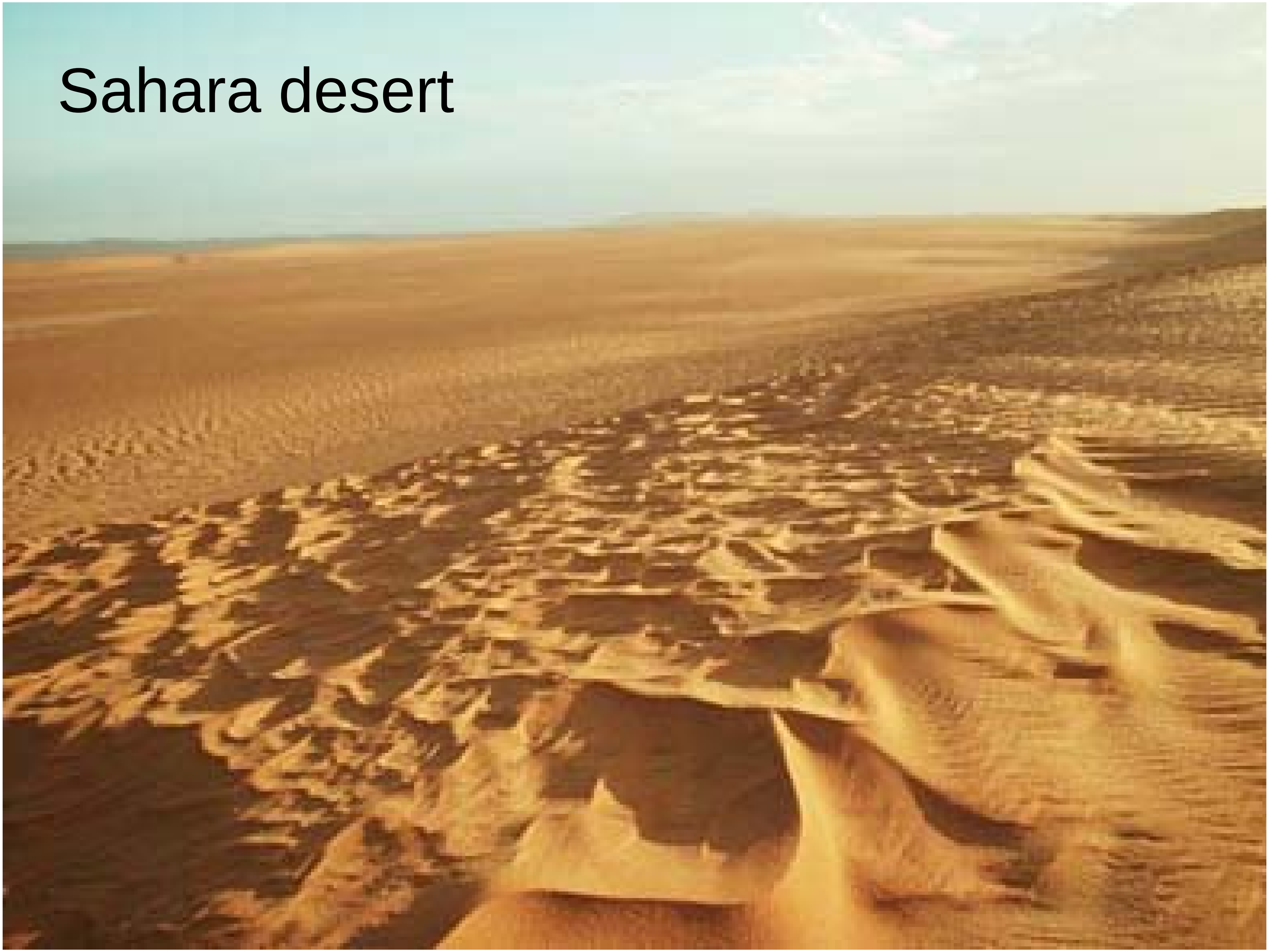






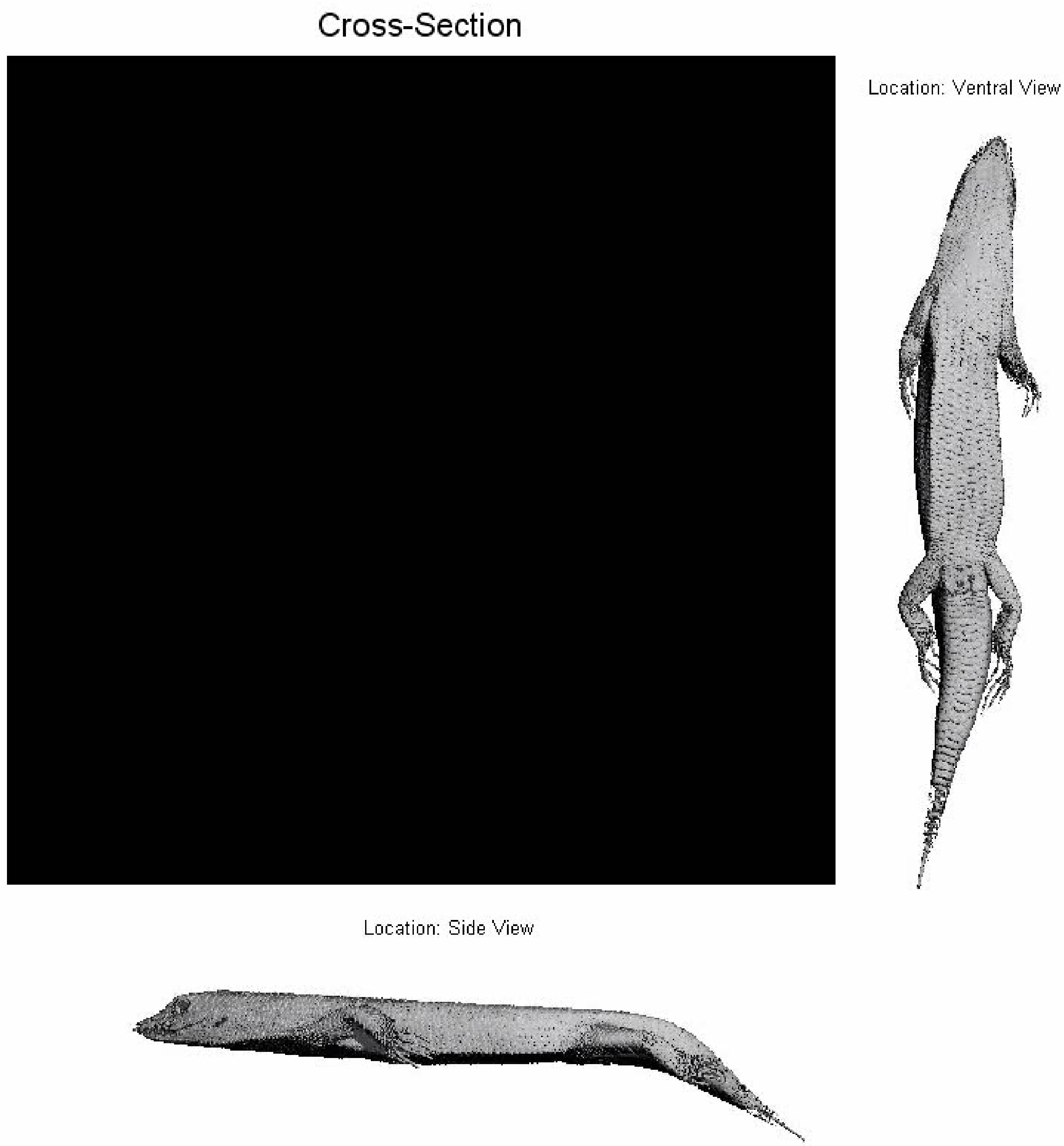





Sahara desert

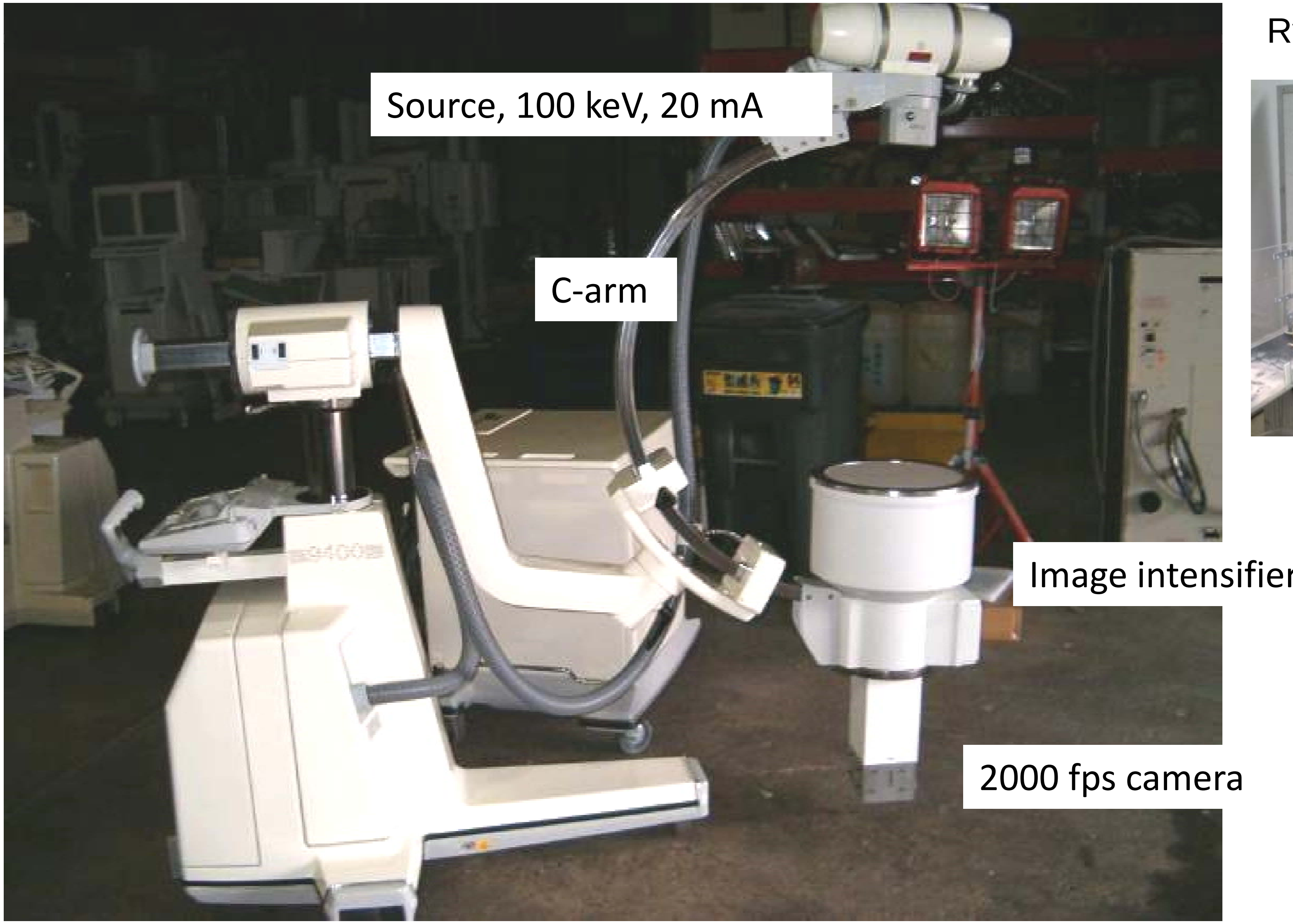
Sandfish lizard (*Scincus scincus*)

5 cm



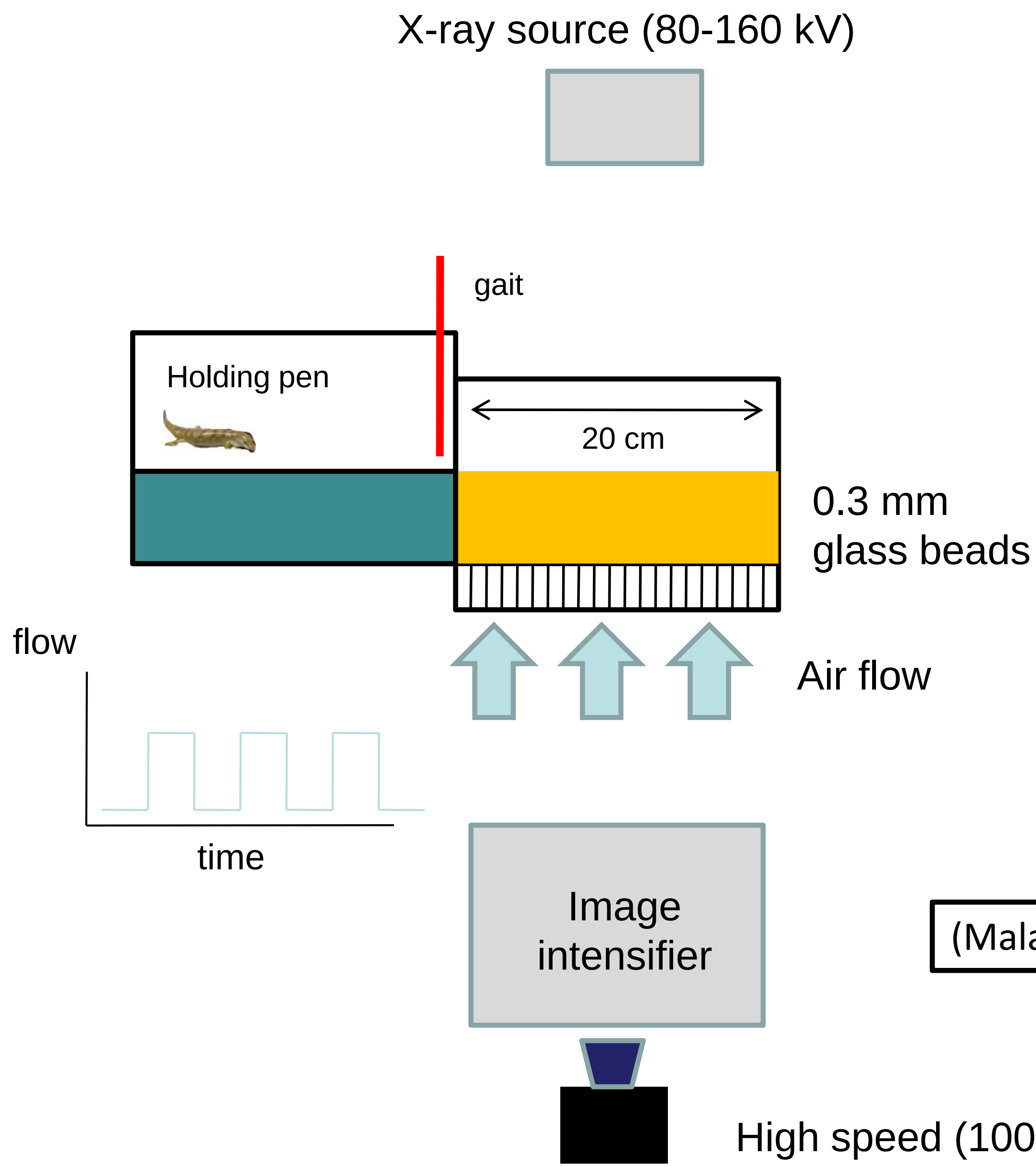
Taken by Sarah Steinmetz at GT micro-CT facility, with Prof. Bob Guldberg,

X-ray imaging of subsurface swimming



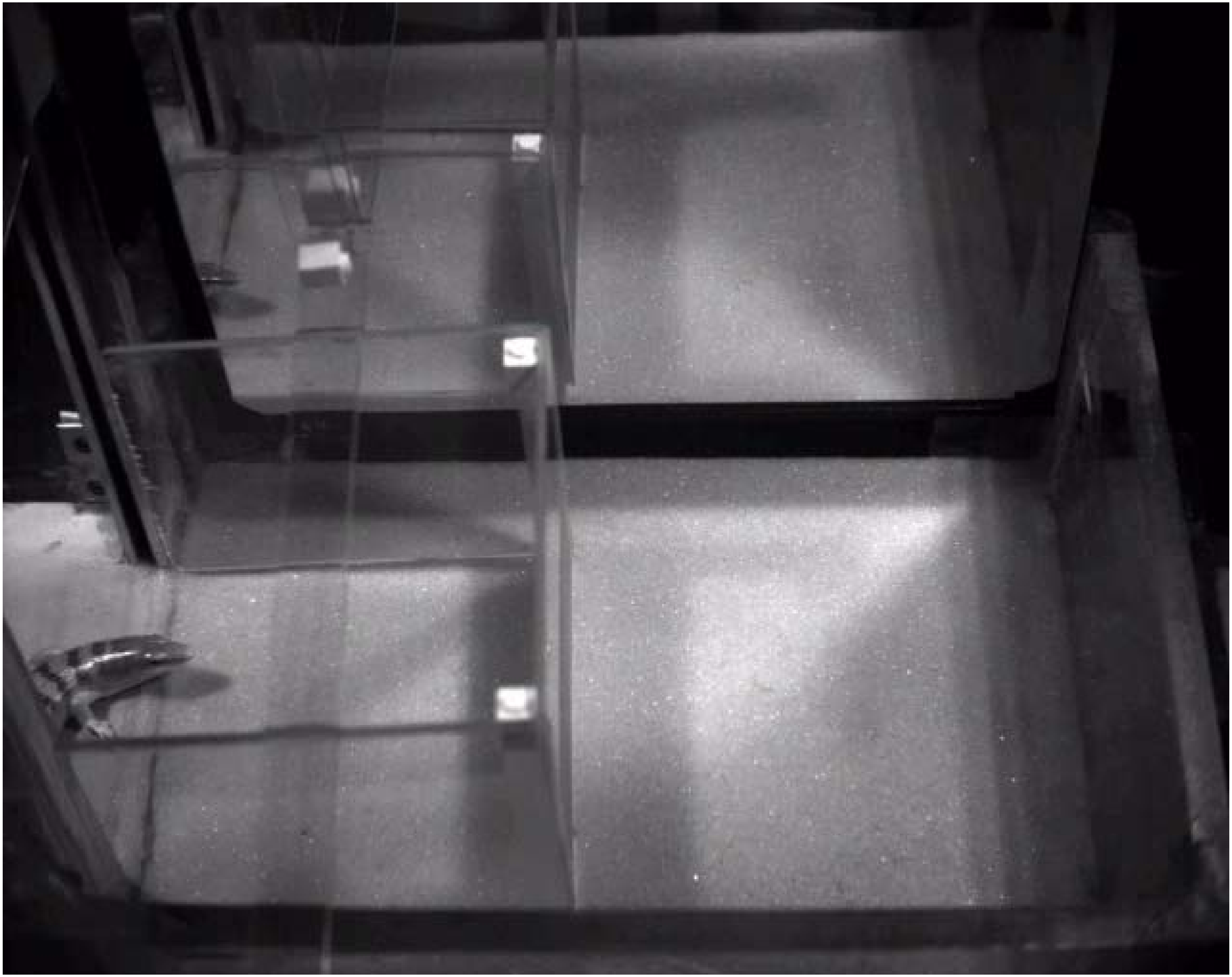
Ryan Maladen

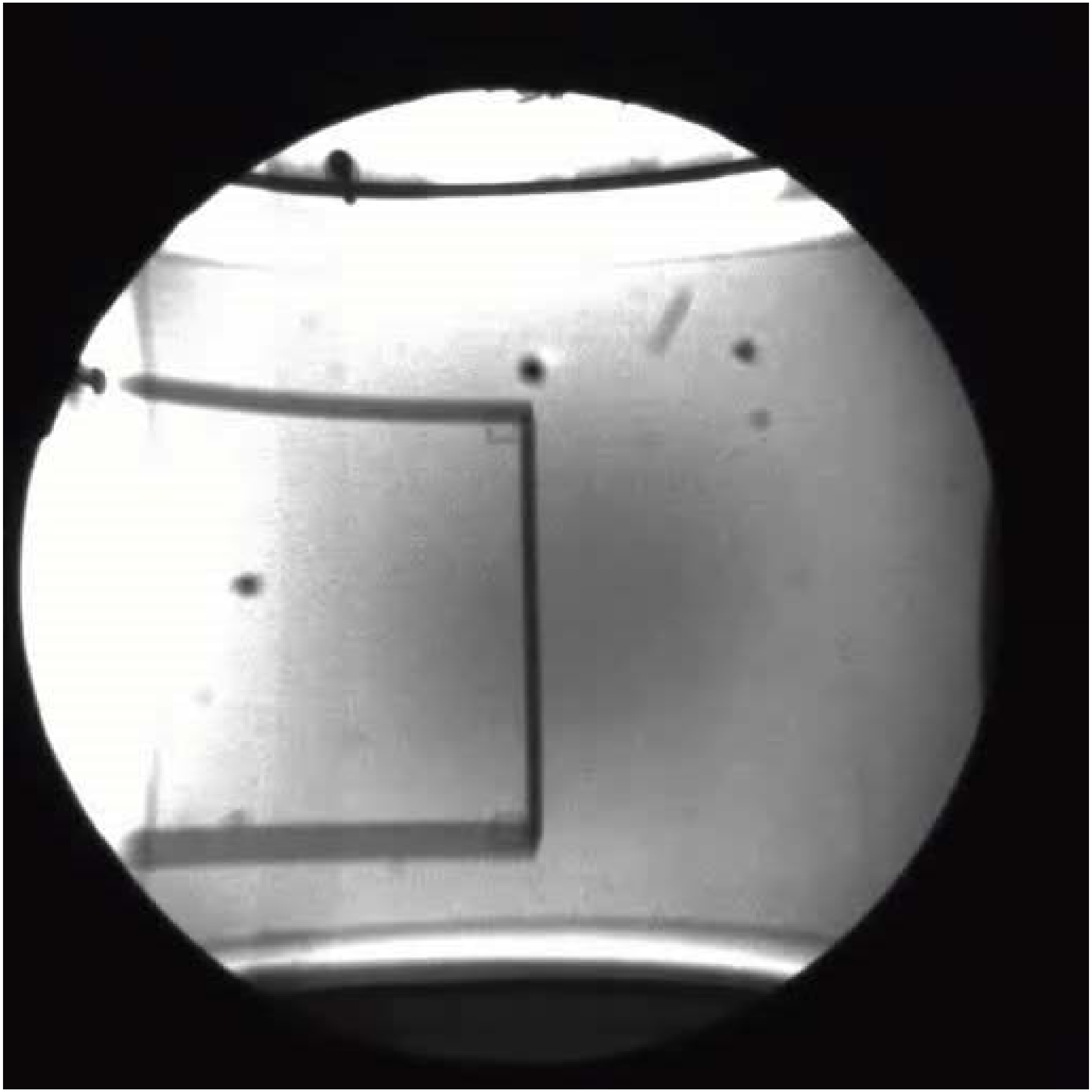
Experimental setup



Air pulses to the fluidized bed sets initial volume fraction 0.58<ϕ<0.63 (doubles resistance forces)

(Maladen, Ding, Li, Goldman, *Science*, 2009)

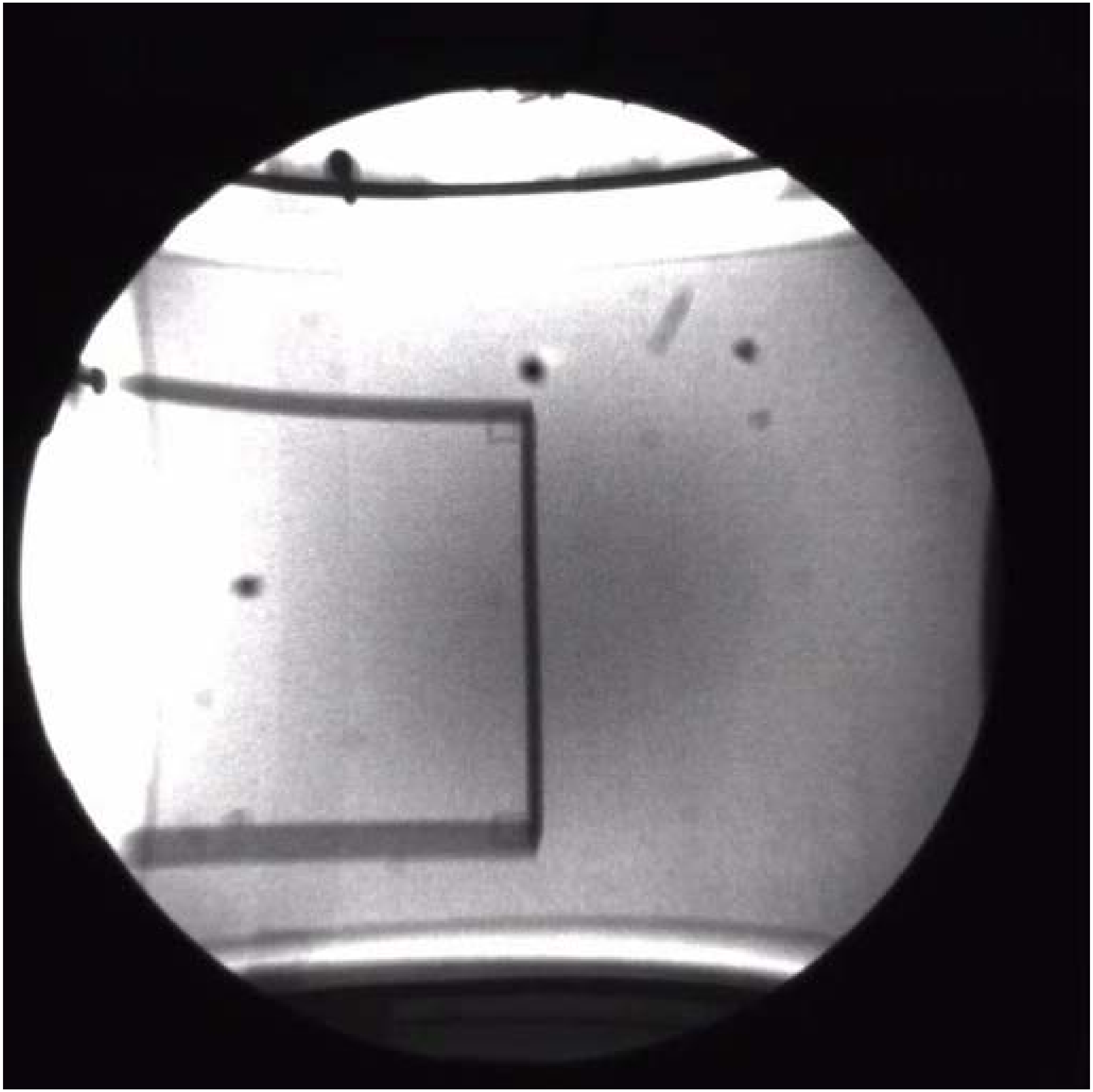




(Maladen, Ding, Li, Goldman, *Science*, 2009)

Real time

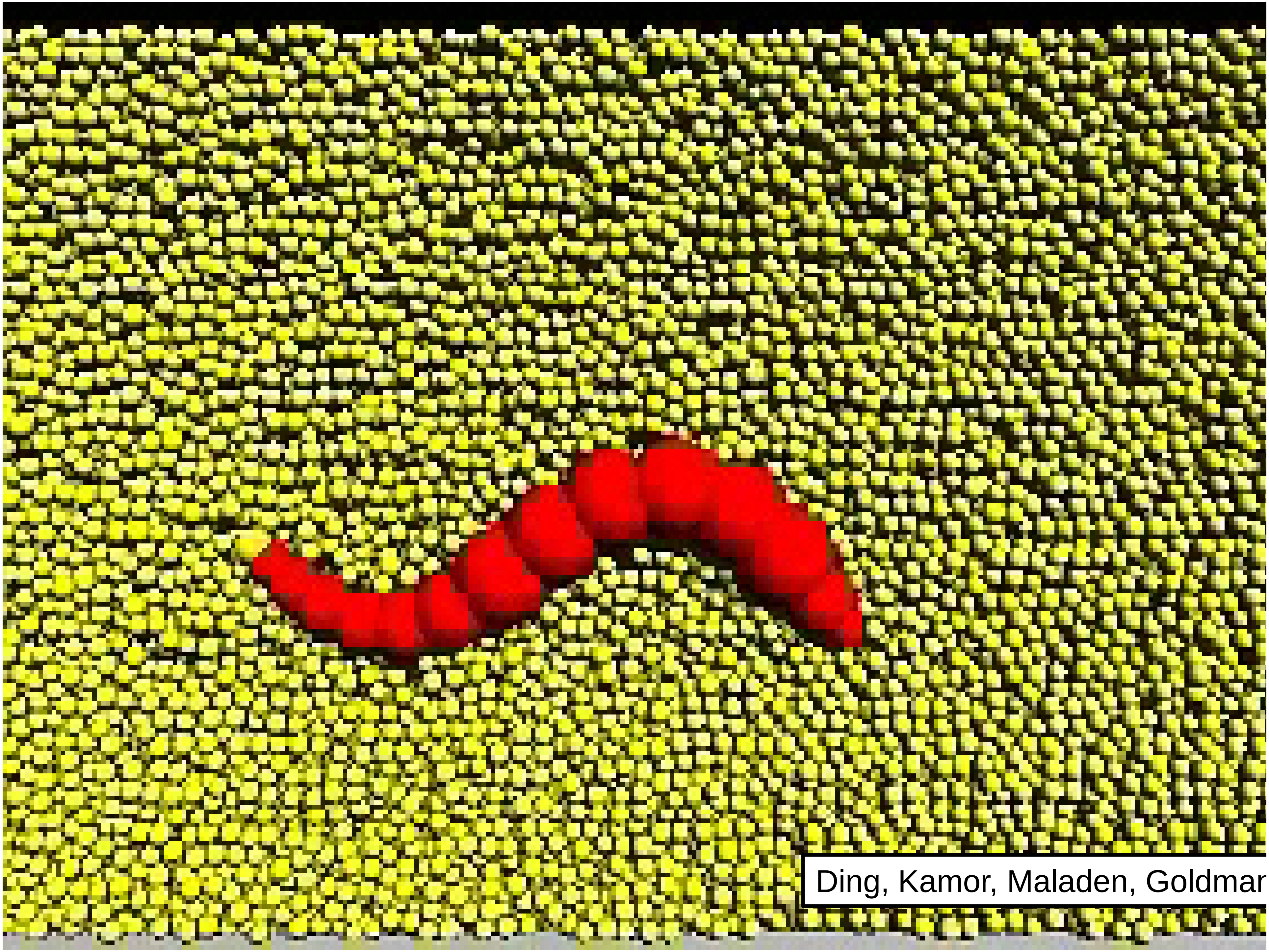
10 cm



(Maladen, Ding, Li, Goldman, *Science*, 2009)

Slowed 10x

10 cm

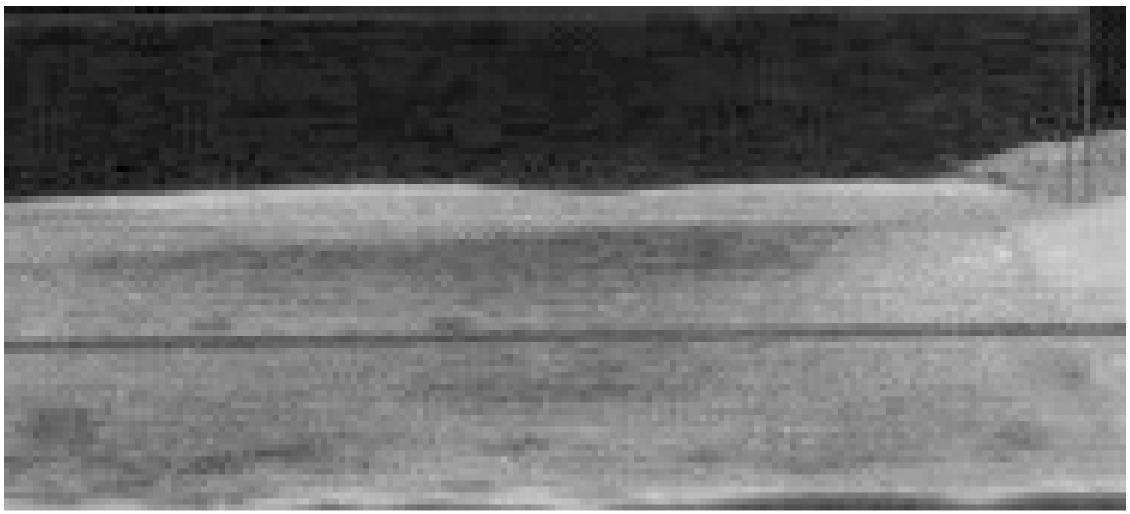

Ding, Kamor, Maladen, Goldman

Terrestrial biomechanics

Alexander, 2005, Cavagna, 1977, McMahon 1980, Blickhan & Full 1989, ….

Major progress in terrestrial locomotion has been made by reducing complexity of foot-substrate interaction

Experiments: foot contact occurs on level, rigid, high-friction trackways



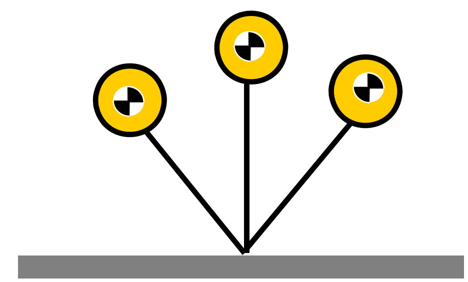
10 cm

Video slowed 10x

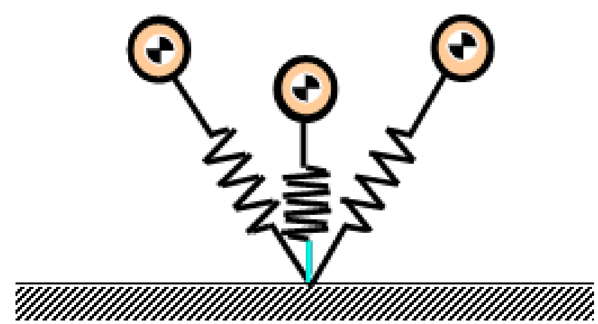
Gecko (*Pachydactylus bibroni*) running on sandpaper covered board at 1 m/sec

Models: point contact is level, rigid & no-slip

Walking: inverted pendulum



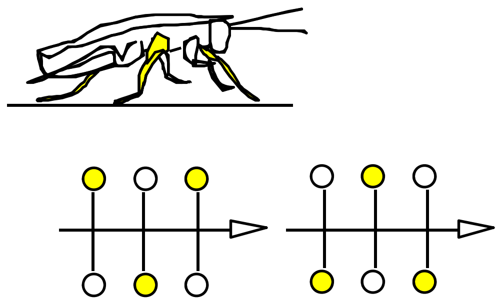
Running: Pogo-stick



Vertical oscillation during rapid locomotion

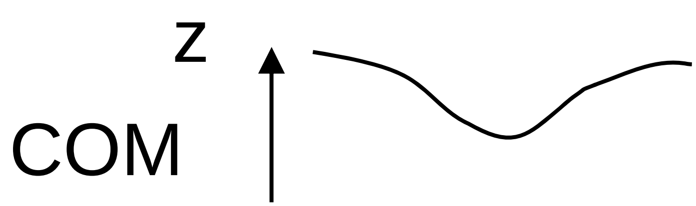
On rigid, level surface with good traction, all animals bounce when they run, trot, or hop

SIX-Legged

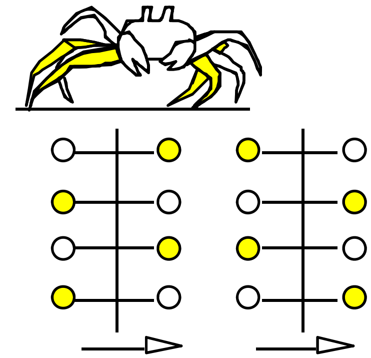


Cockroach

Cockroach: 0.3 mm

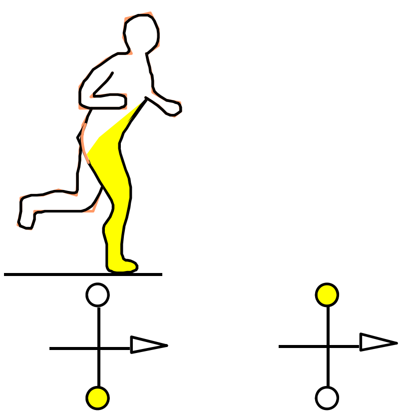


EIGHT- Legged



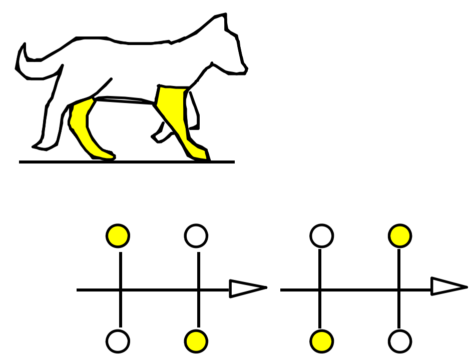
Crab

TWO-Legged



Human

FOUR-Legged

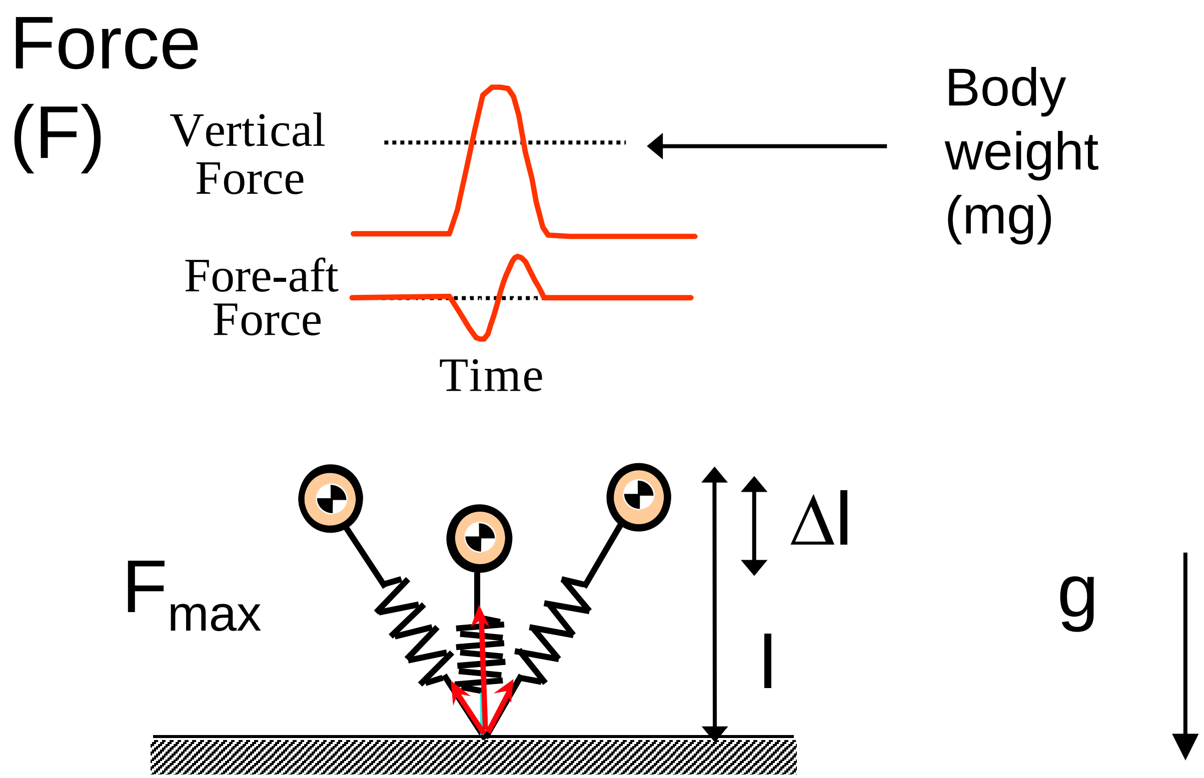


Dog

Force pattern for COM independent of morphology

Blickhan, *J. Biomechanics*, 1989
Blickhan & Full, *J. Comp. Physiol. A*, 1993

Spring
Loaded
Inverted
Pendulum
(SLIP)
model



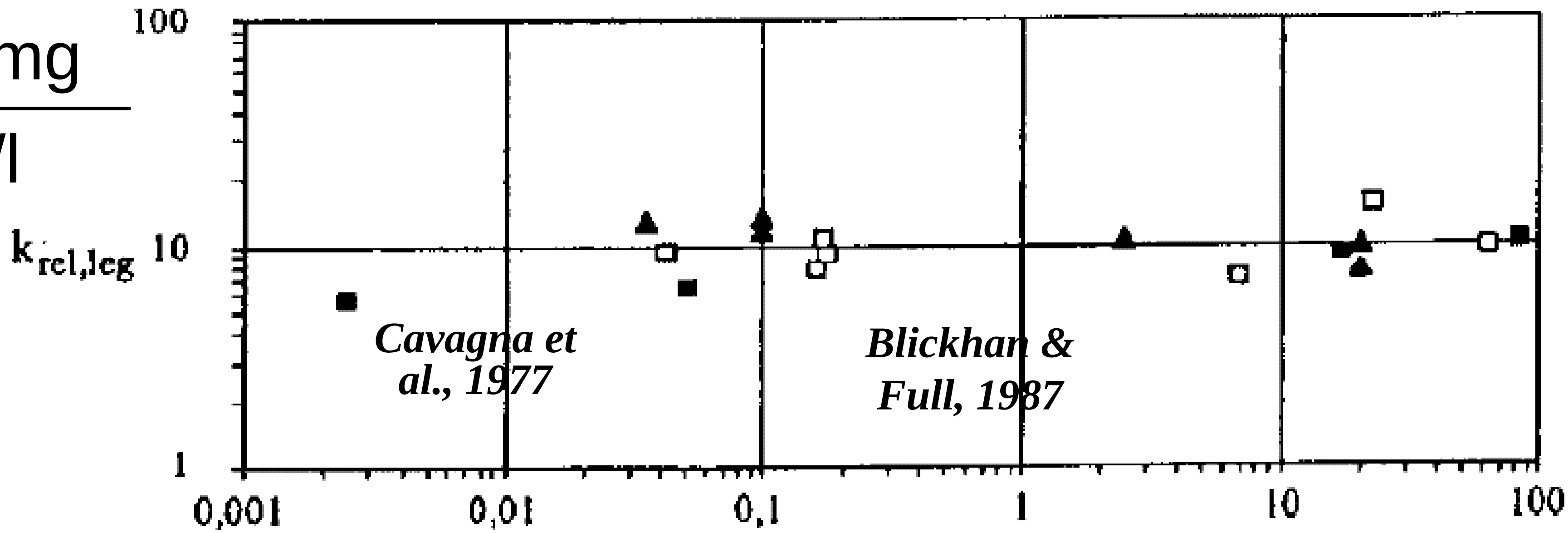
eg, Cockroach: 0.3 mm,10 mN

Unifying principle of terrestrial locomotion on level, flat, rigid ground

Non-dimensional spring constant

Blickhan & Full, *J. Comp. Physiol. A*, 1993

$$\frac{F/mg}{\Delta l/l}$$



Body mass (kg)

○ *TROTTERS*
■ *RUNNERS*
▲ *HOPPERS*

Stability matters

Slowed 30x

Periplaneta americana

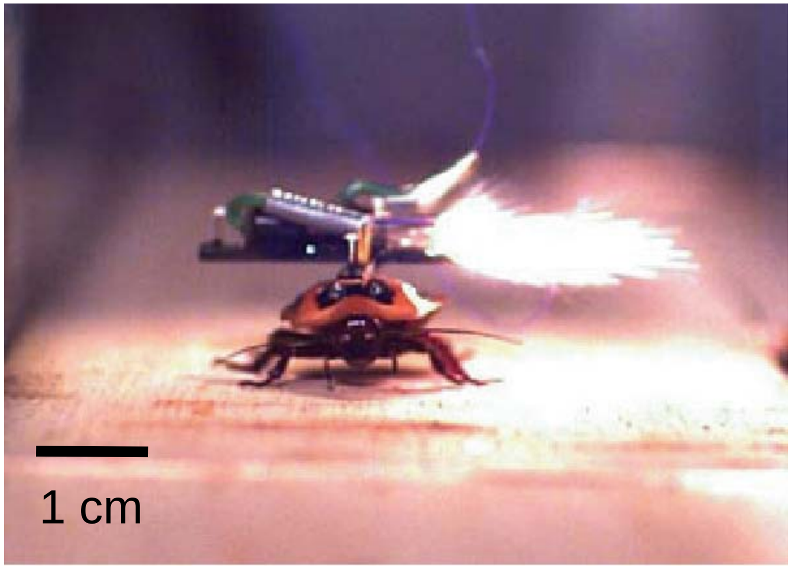
1 cm

sandpaper

graphite coated stainless steel

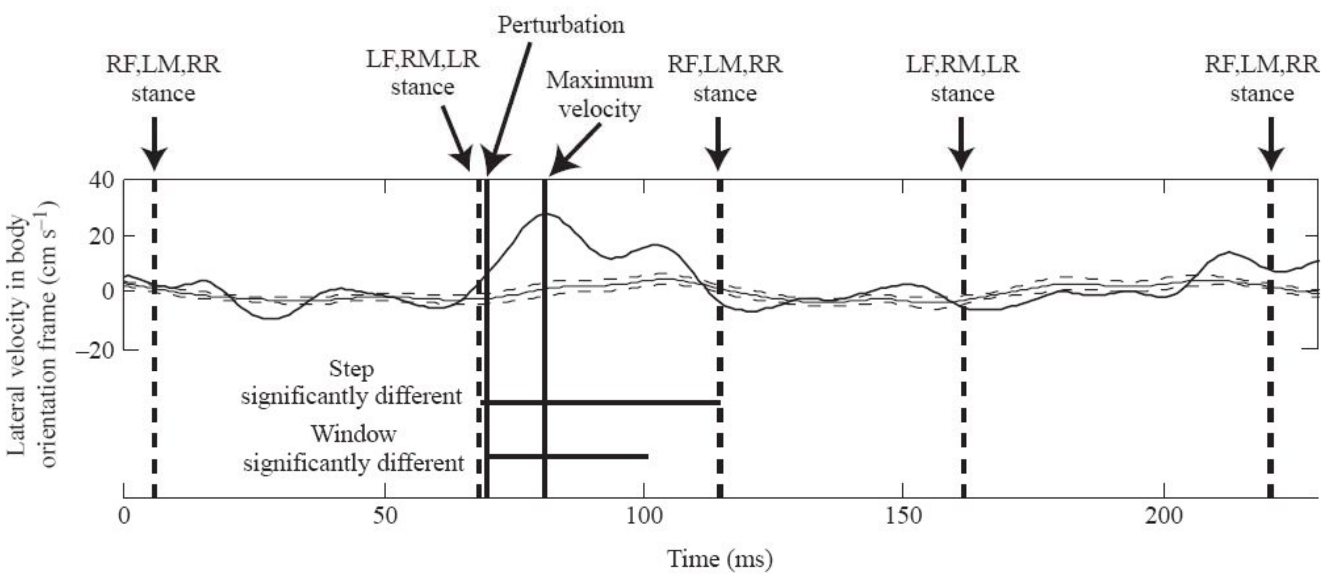
Rapid Stabilization

Jindrich, Full JEB (2002)

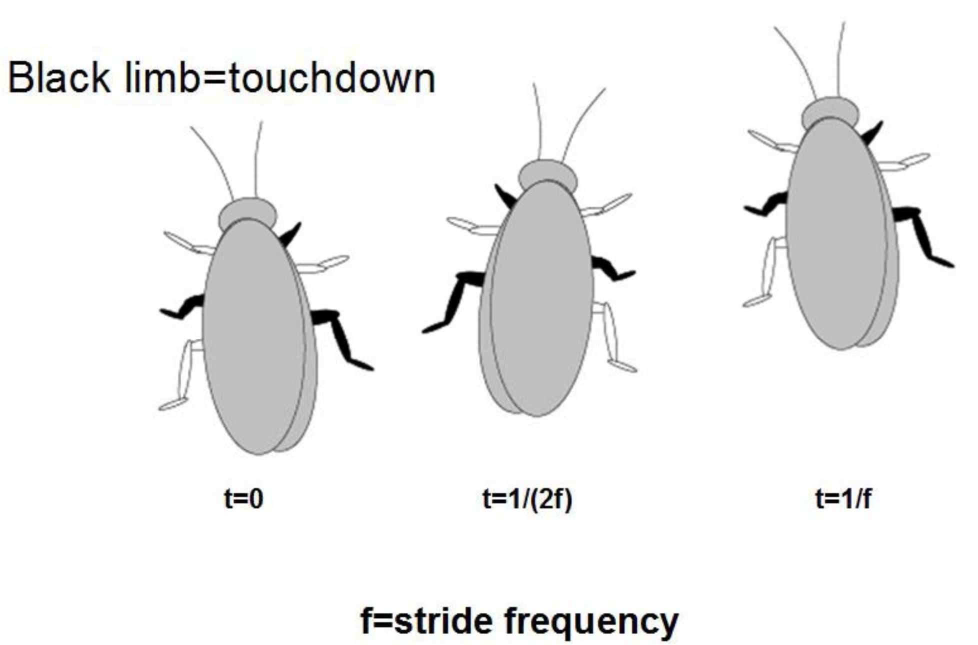


On level, rigid, no-slip ground, give large perturbation:

Recovery in less than two steps (<50 msec), challenging the fastest neural reflexes



Alternating tripod gait

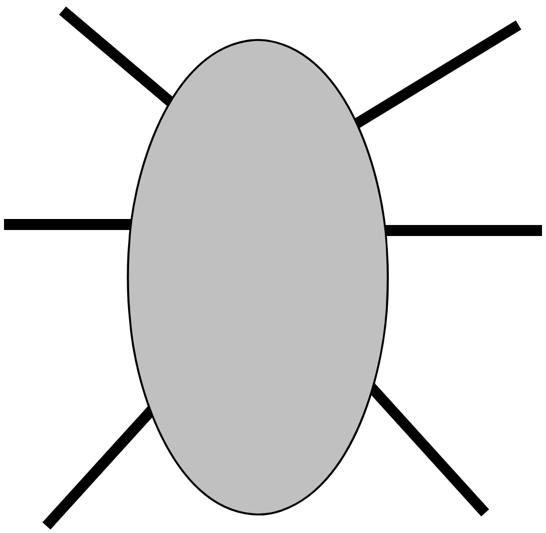


- Three legs fire in synchrony
- Used at fast speeds (>20 cm/sec)

Slowed 20x

2 cm

75 % of all species of animals have hexapod body plan and use alternating tripod gait



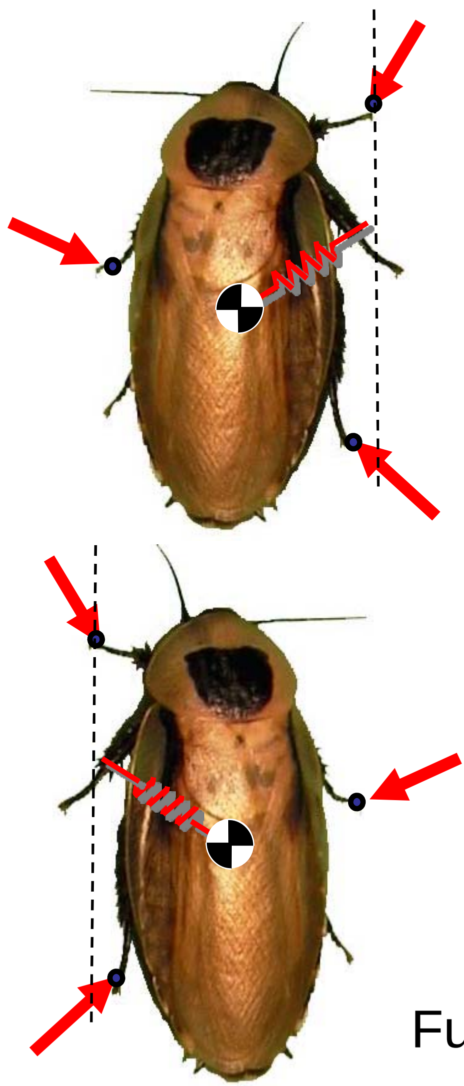
Insect Diversity

There are currently roughly 1,032,000 described species of animals on the planet earth. The total number may be much higher, as much as 10-30 million in fact. Of those described species, insects comprise 751,000 species or 72.8% of all the animals. Insects are an even larger proportion of the terrestrial animals (nearly 85%). One of the major features of insects is their extraordinary diversity in terms of numbers and morphological forms. It is important to have a general understanding of the major groups (Orders) of insects and in two lectures I will present an overview of the 32 currently recognized insect orders. We will do this in terms of a timeline (from the origins of insects in the Devonian [400 million years ago]) to the present. I will present the major evolutionary events in insect diversification, including the origins of wings, the origins of metamorphosis, and the rapid diversification of insects in association with the flowering plants (in the Cretaceous). We will also examine the major extinctions in insects, including the end-Permian and end-Cretaceous extinctions. You will come away with an understanding of both the timeline of insect evolution as well as the diversity of the insect orders.

From *Alien Empire* course at Cornell , Entomology 201

Modeling lateral stability

Schmitt & Holmes, *Biological Cybernetics*, 2000

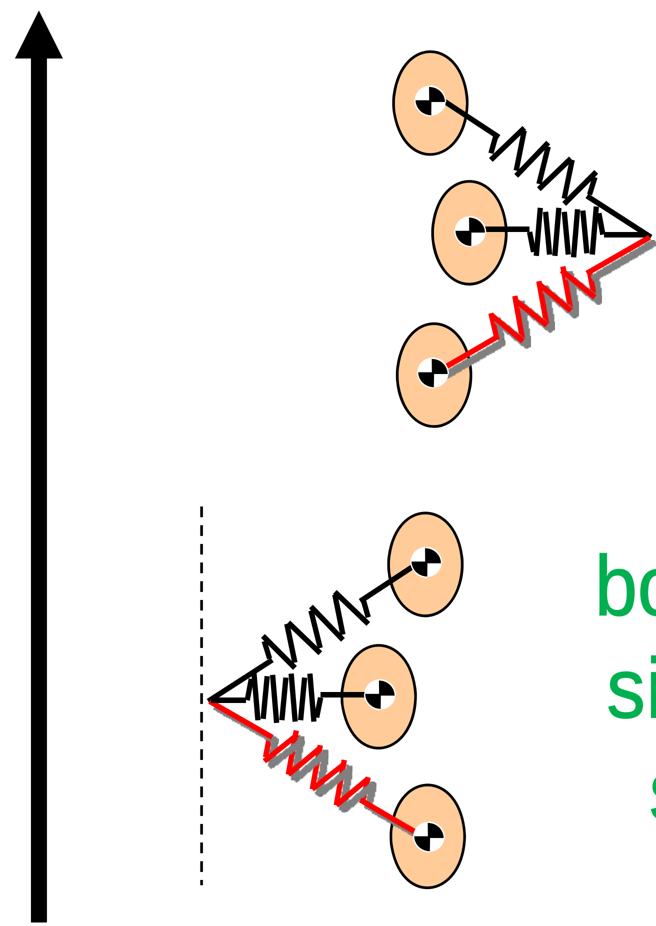


Level Running

3 Legs Acting as One

Full & Tu, 1990

Lateral Leg Spring Model



bounce side to side

Step-to-step return map

Schmitt & Holmes, *Biological Cybernetics*, 2000
Schmitt, Garcia, Razo, Holmes & Full, *Biological Cybernetics,* 2002

Equations of motion of body

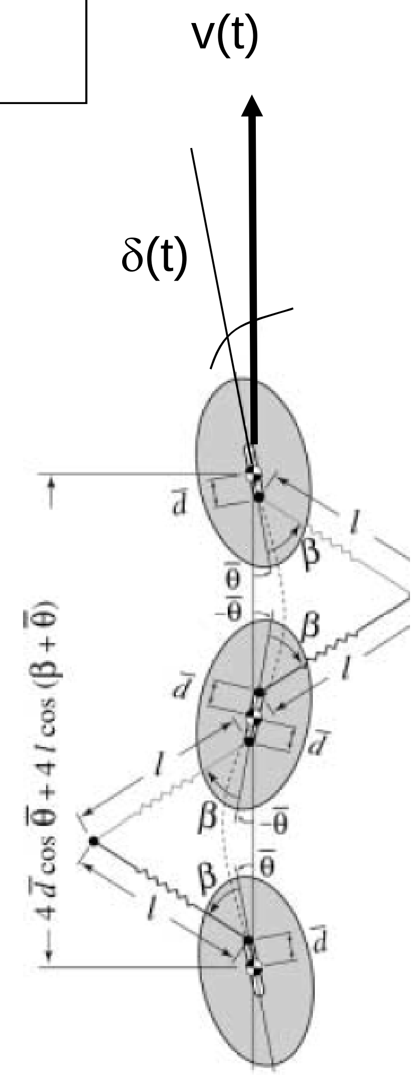
$$m\ddot{\mathbf{r}} = \mathbf{R}(\theta(t))\mathbf{f}, \quad I\ddot{\theta} = (\mathbf{r}_F(t_n) - \mathbf{r}) \times \mathbf{R}(\theta(t))\mathbf{f} \ ,$$

With R, the rotation matrix needed to transform foot forces to body coordinates, f the leg forces, r the touchdown foot position.

Integrate these on a step by step basis, obtain Poincare map F that takes,

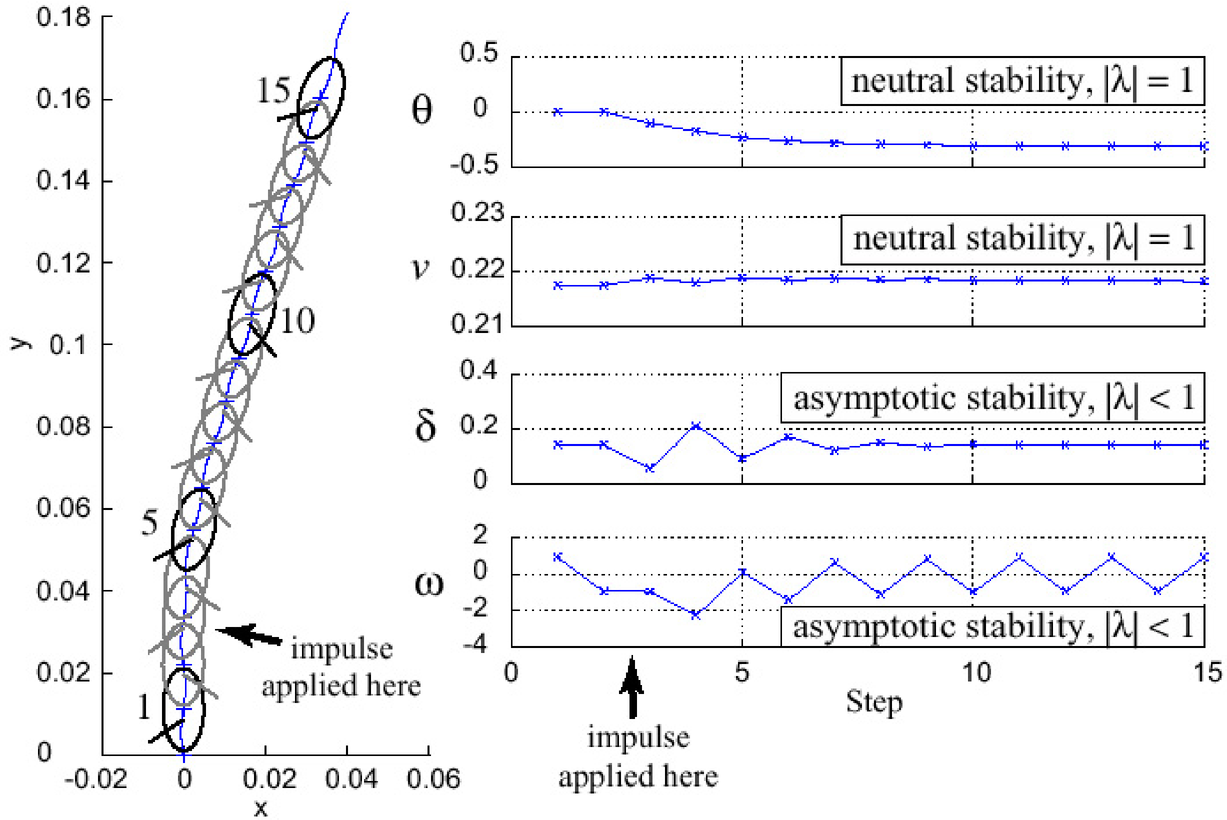
$$(v_{n+1}, \delta_{n+1}, \theta_{n+1}, \omega_{n+1}) = \mathbf{F}(v_n, \delta_n, \theta_n, \omega_n) \ ,$$

Where
v is forward velocity
δ is heading relative to velocity of COM
θ is angle of body in world frame
ω is $d\theta/dt$



Tuned spring leg & non-holonomic foot constraints: asymptotic stability

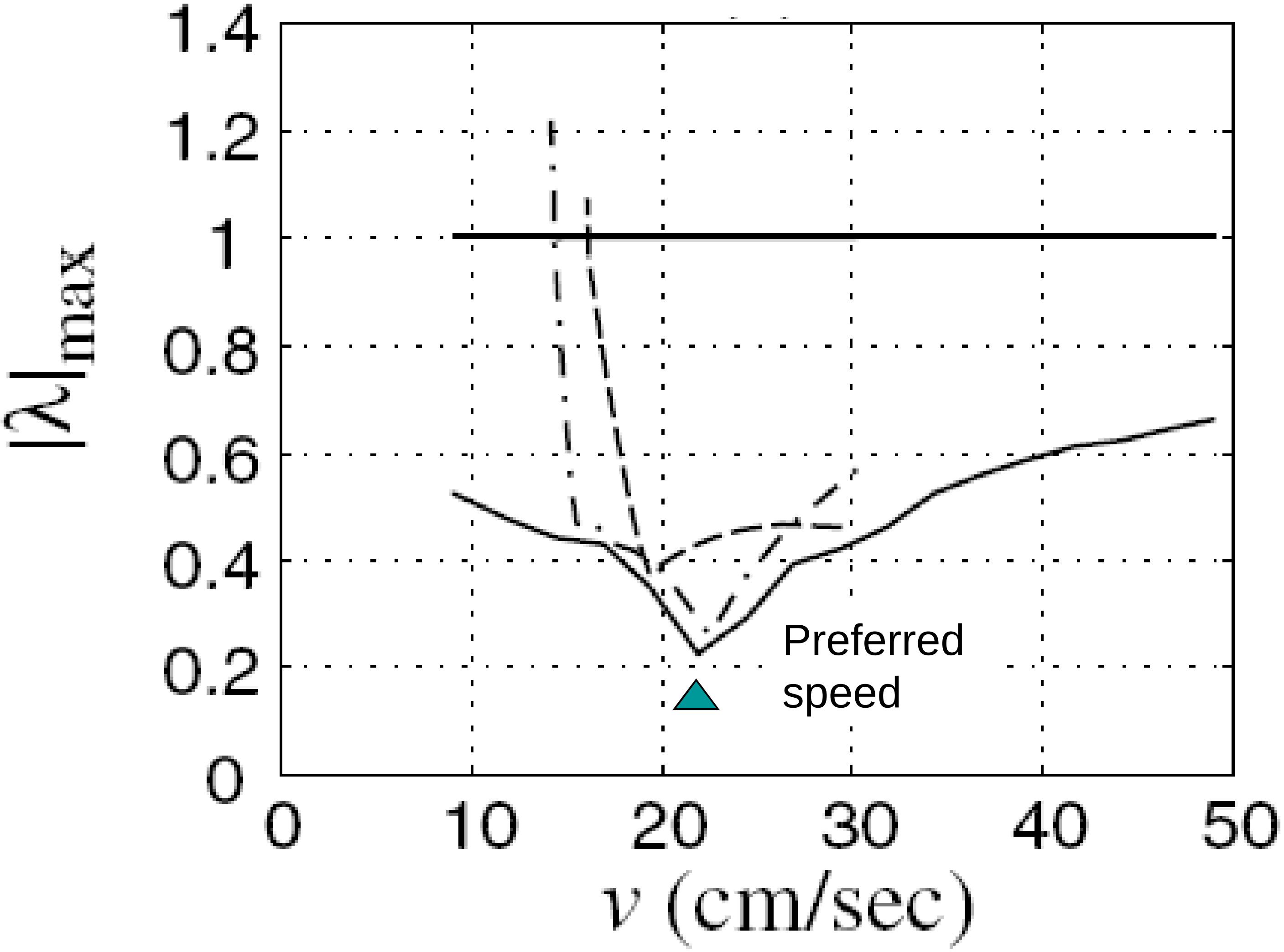
Schmitt & Holmes, *Biological Cybernetics*, 2000
Schmitt, Garcia, Razo, Holmes & Full, *Biological Cybernetics,* 2002



LLS model yields rapid stable response to perturbation--TURN OFF THE BRAIN?

LLS model predicts preferred speed

Schmitt, Garcia, Razo, Holmes & Full, *Biological Cybernetics,* 2002



λ are the eigenvalues of the linearized step-to-step map F

Simple mechanical control of a robot

Journal paper: Saranli, Buehler & Koditschek, *Int. J. Rob. Res.*, 2001

R-Hex: Robot Hexapod 5 kg, 4 m/sec

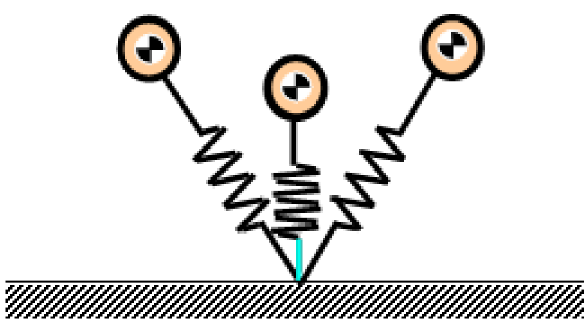
Implement SLIP+LLS in a physical device

Templates for locomotion

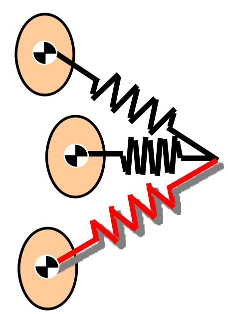
Full & Koditschek, JEB, 1999

Template: simple dynamics such that diversity of organisms during a behavior 1) control limb forces to 2) target this dynamics which 3) simplifies control of the behavior

Running: SLIP



Running (horizontal plane): Lateral Leg Spring

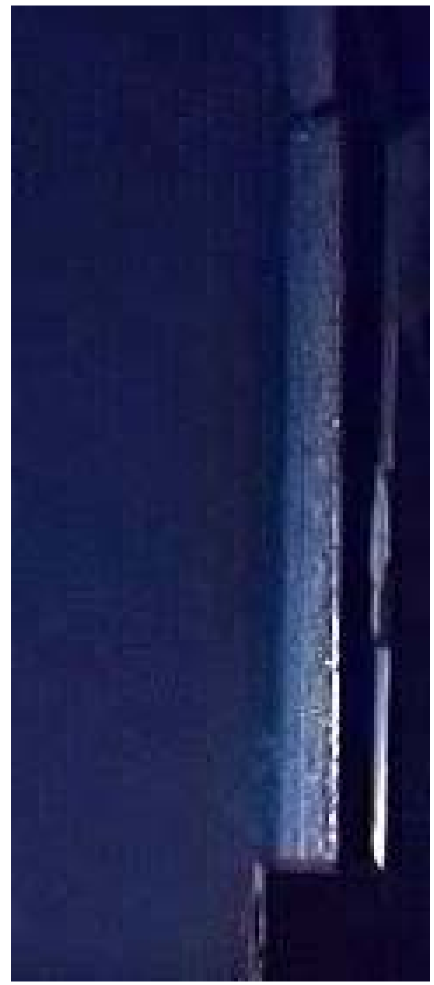


Do organisms achieve rapid stable vertical locomotion using a *template*?

Gecko lizard

Cockroach

10 cm



50 cm/sec,
30 steps/sec

2 grams

Hemidactylus garnoti

Periplaneta americana

1 gram

No vertical climbing templates yet

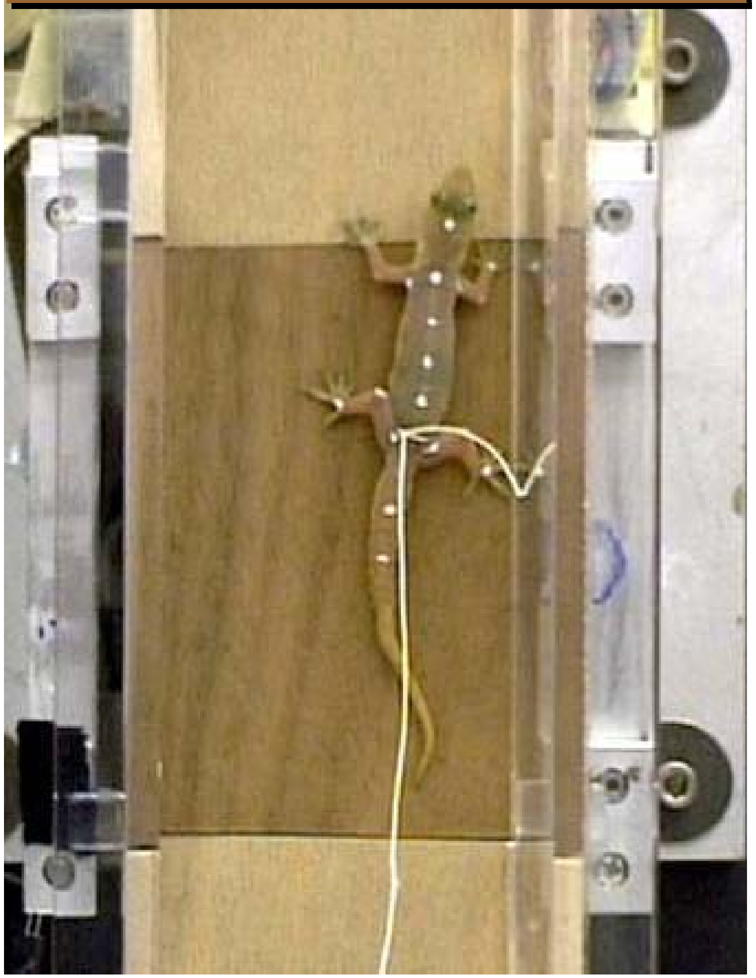
R-Hex at Southwest Research Institute, 2001

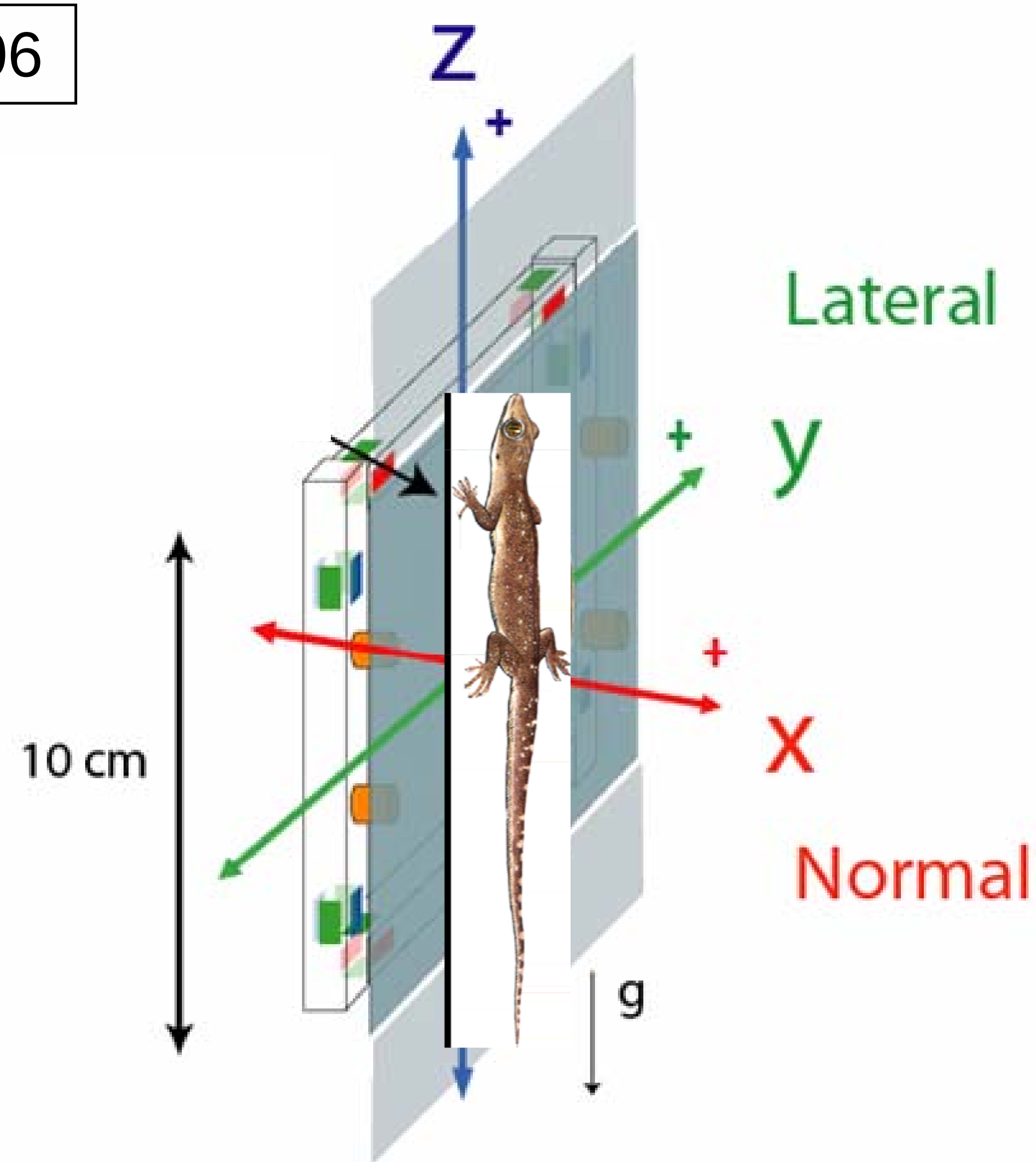
(Koditschek, Buehler, Rizzi, Full)

Wall Reaction Force Measurement

Autumn et al, *J. Exp. Biol.* 2006

3D Force Platform





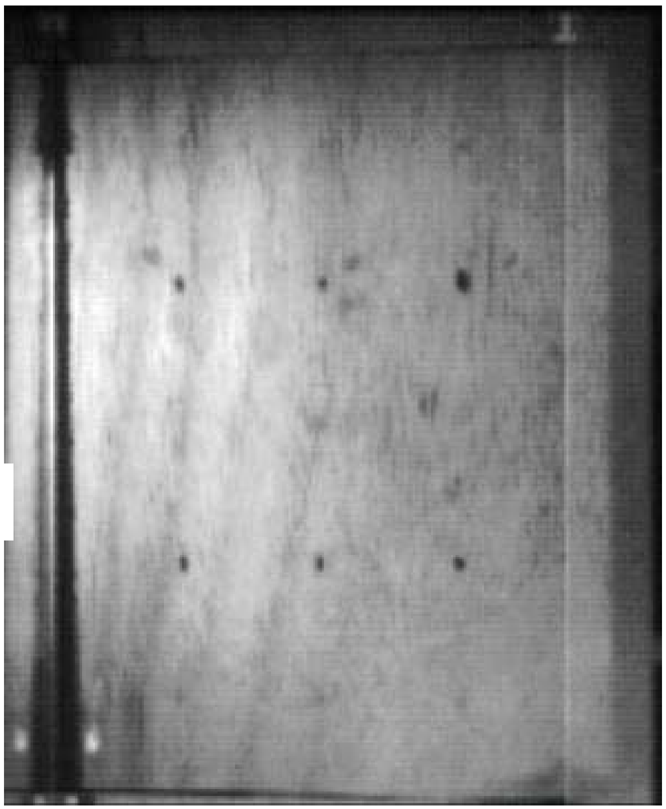
Sensitive to 0.2 mN

Gecko climbing dynamics

Autumn *et al.* 2006

Steady-state trot on smooth balsa, slowed 20x

10 cm



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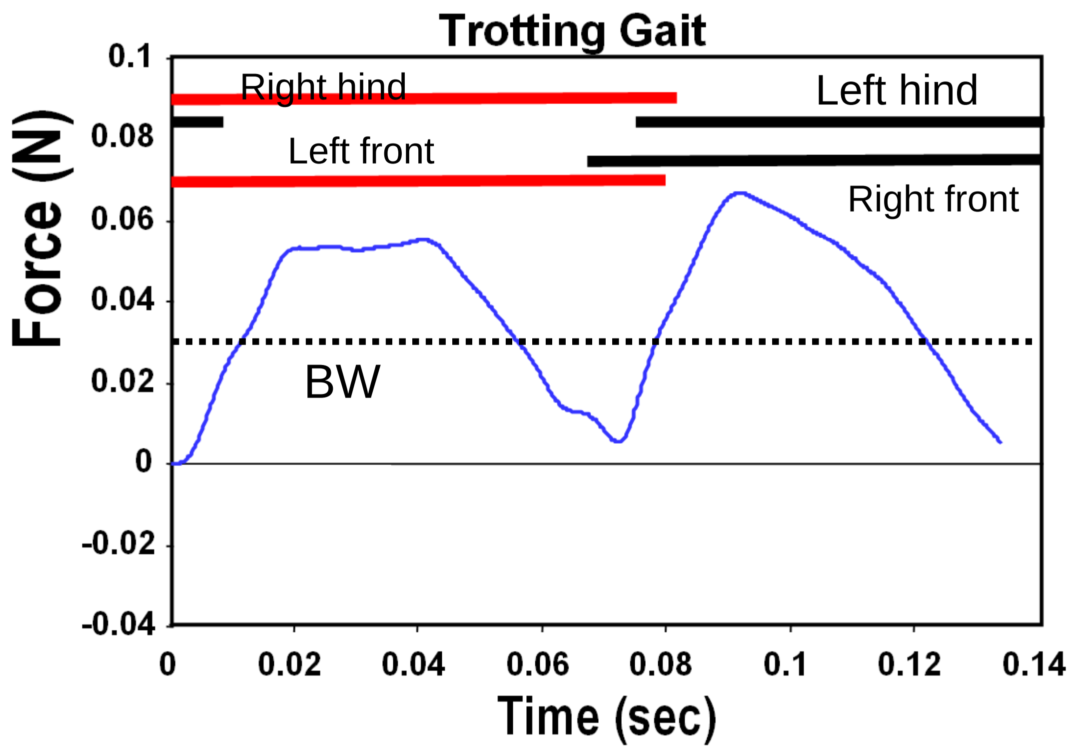
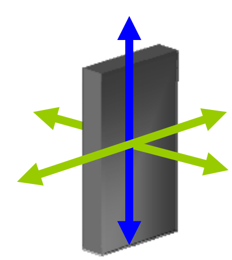
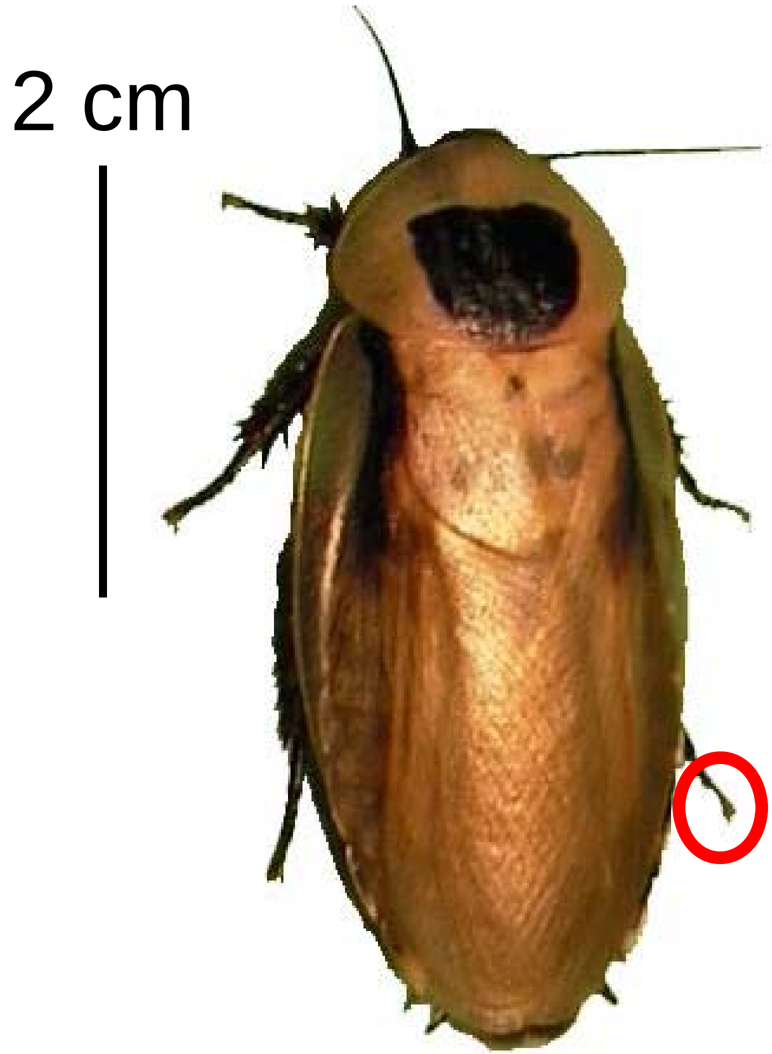
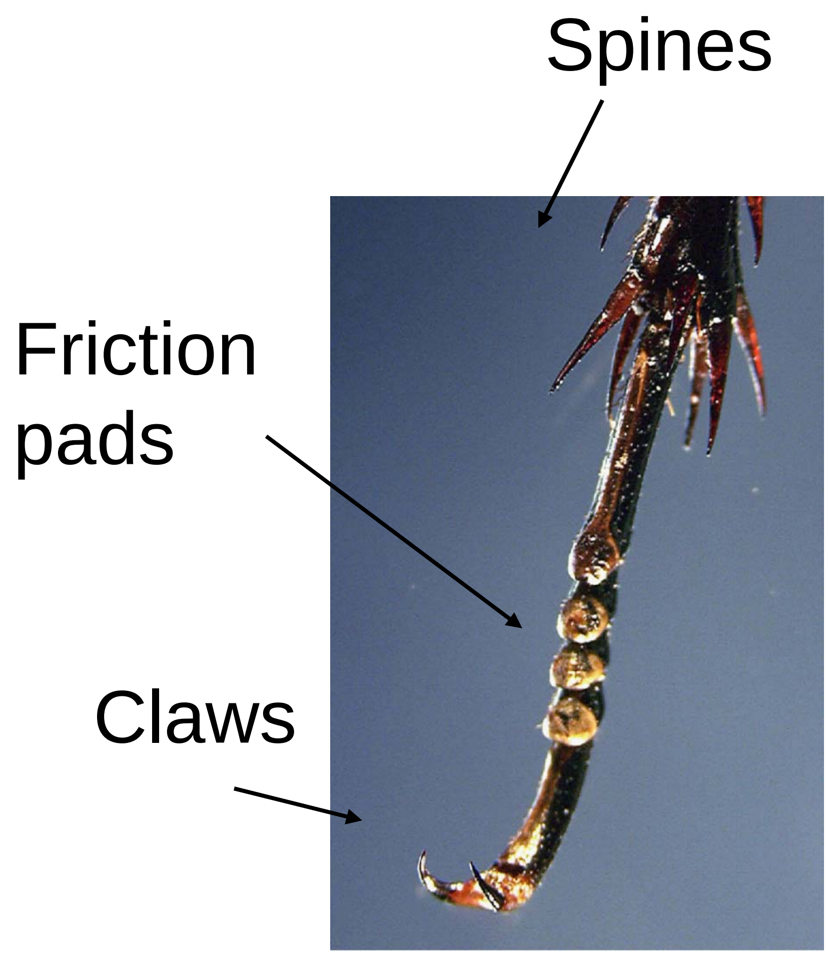
Cyclically accelerate the body upward



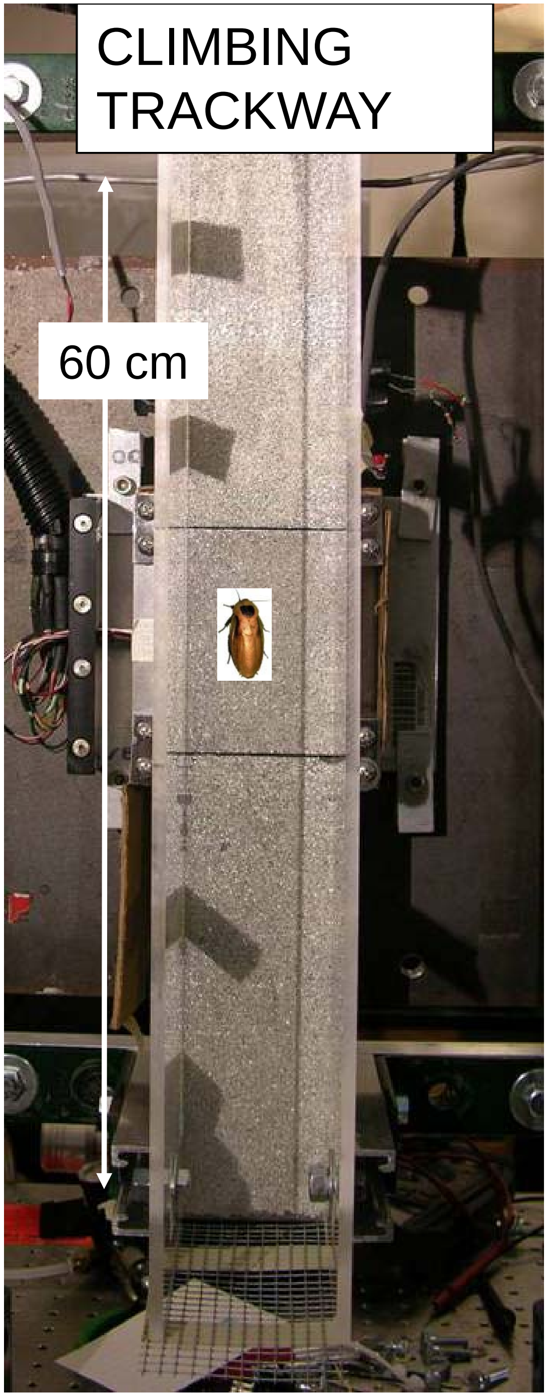


Large rapidly climbing insect:
Death's head cockroach



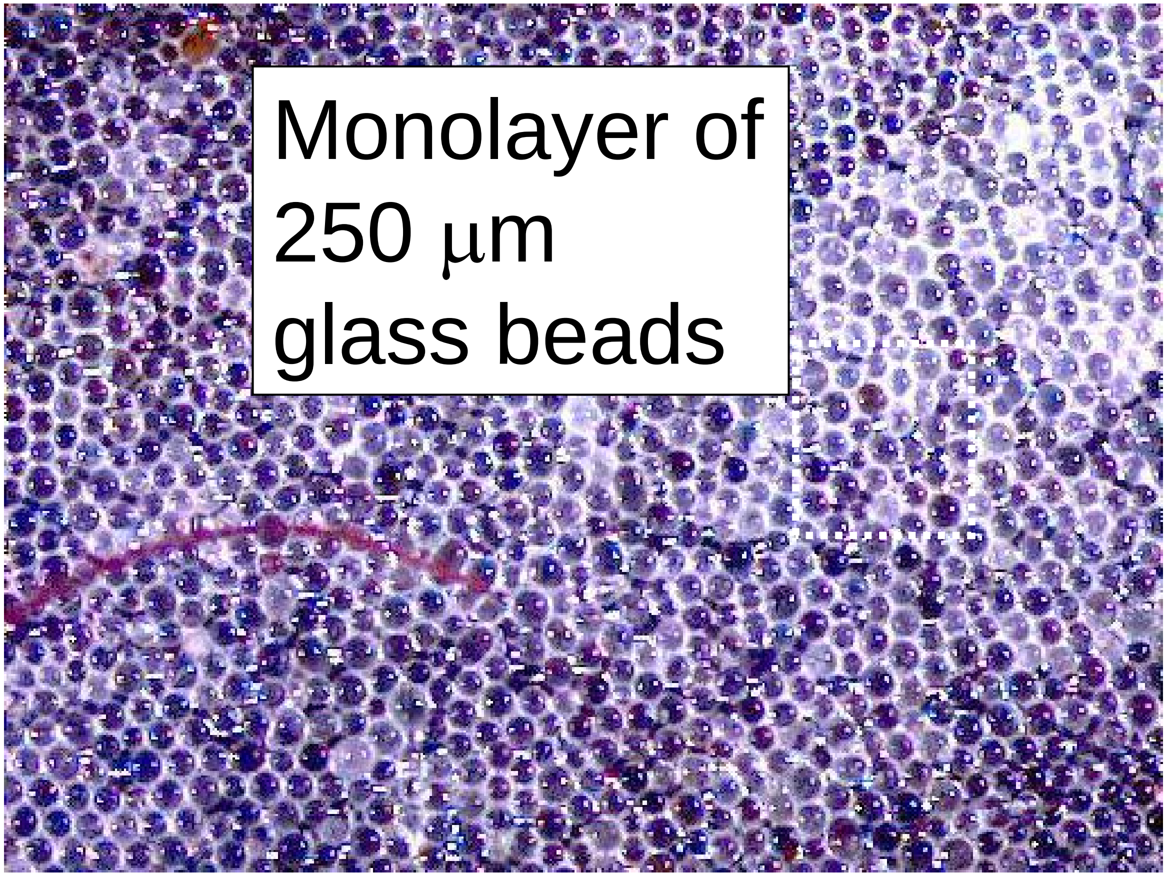


Courtesy R.J.Full



Controllable climbing trackway

Goldman, Chen, Dudek & Full, *J. Exp. Biol.* 2006



Slowed 20x

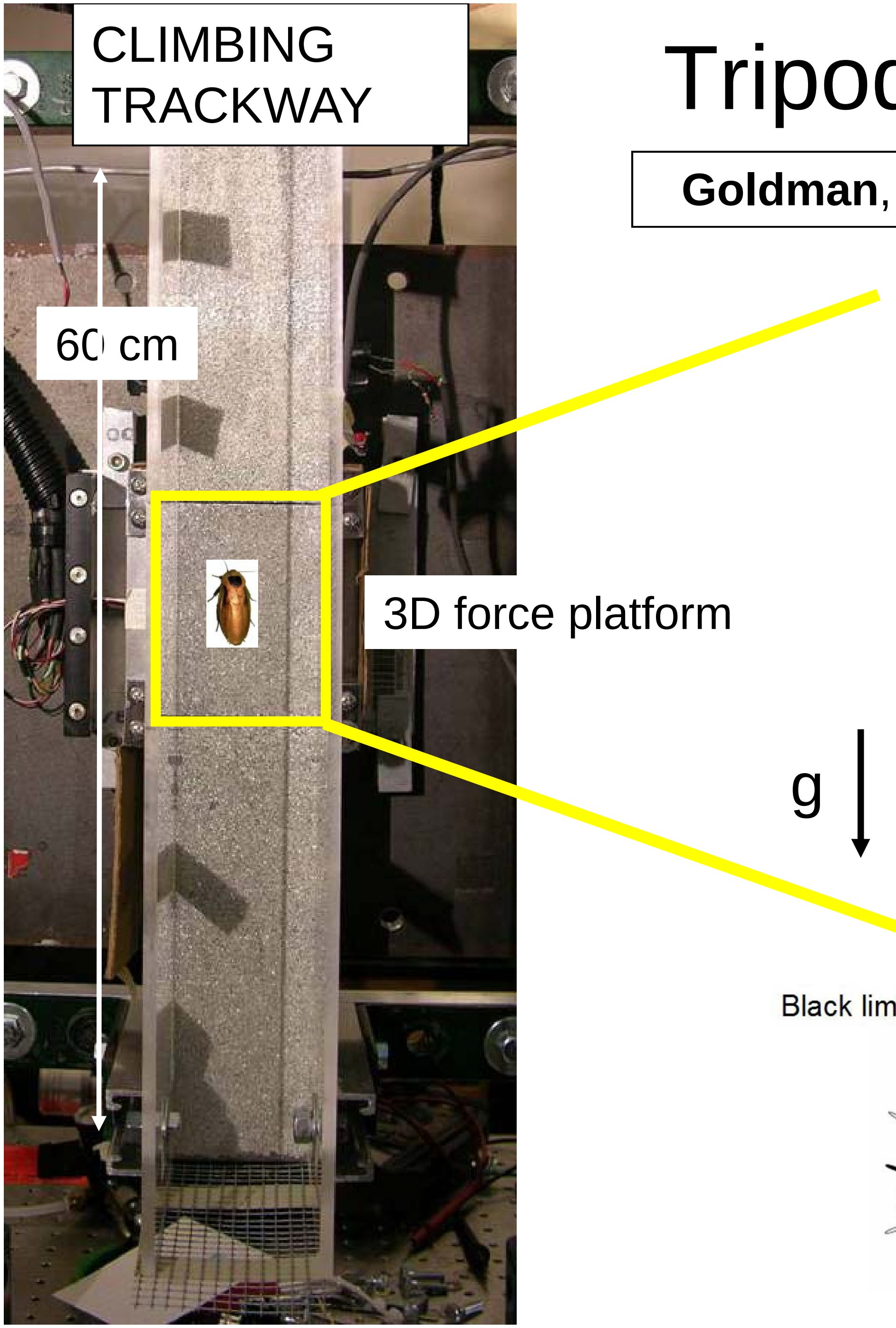
Blaberus discoidalis climbing at 30 cm/sec

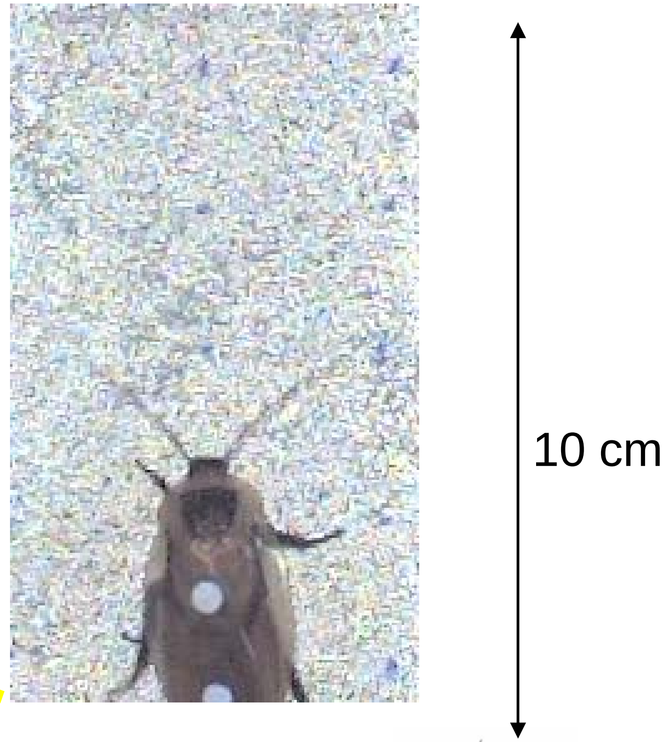
3 grams

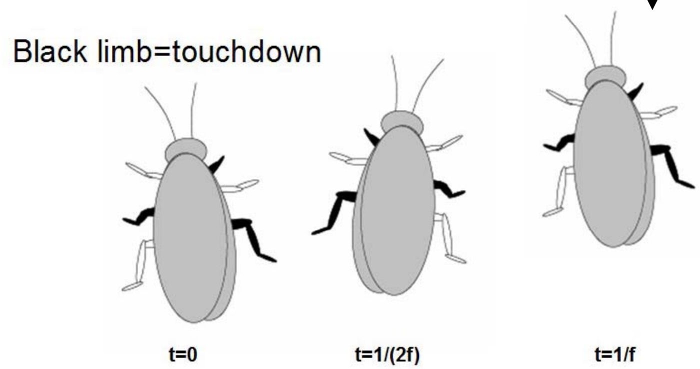
10 cm

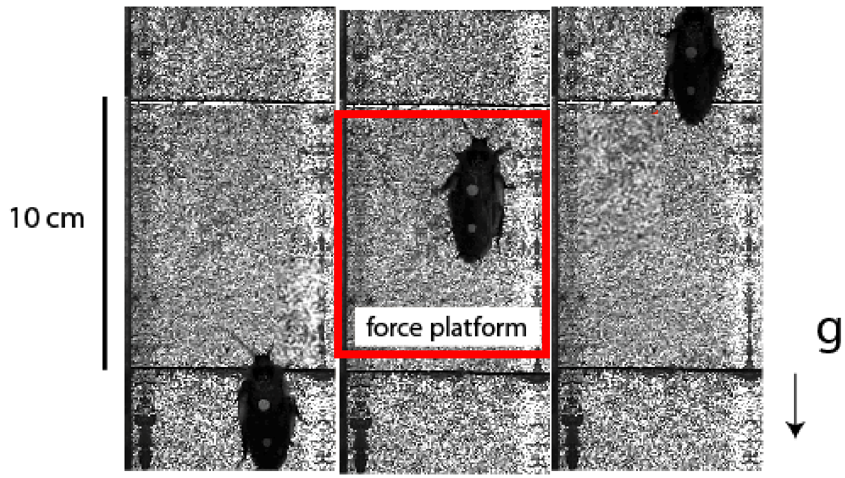
Tripod gait during climbing

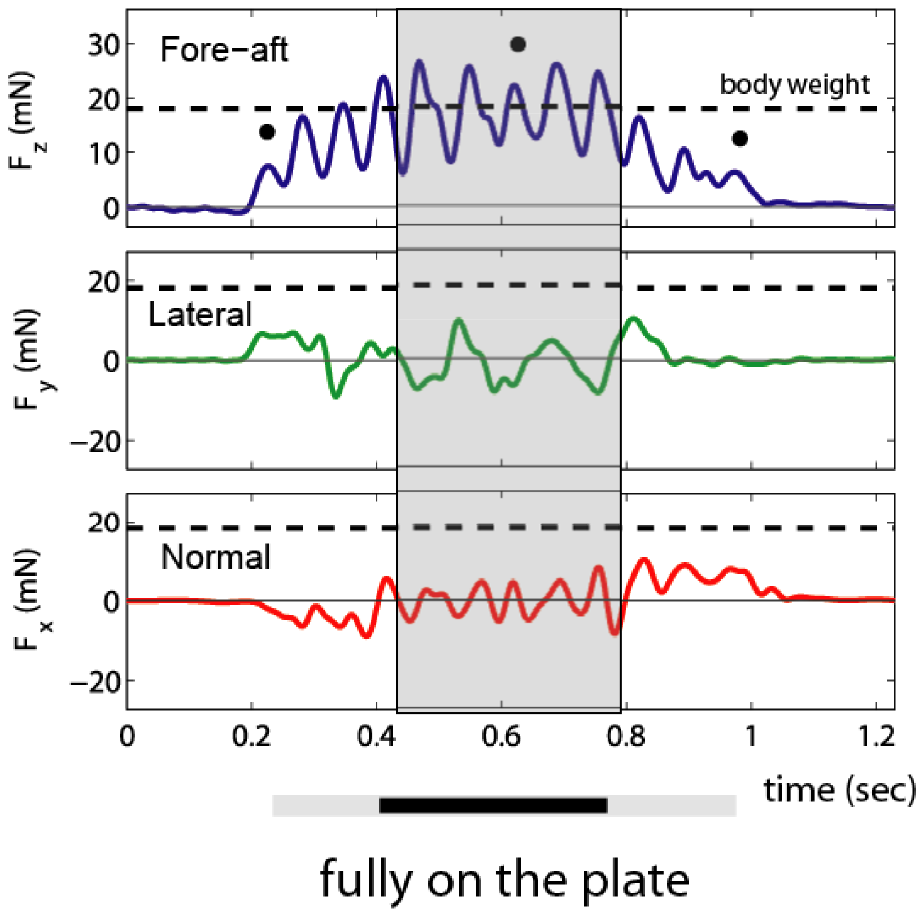
Goldman, Chen, Dudek & Full, *J. Exp. Biol.* 2006







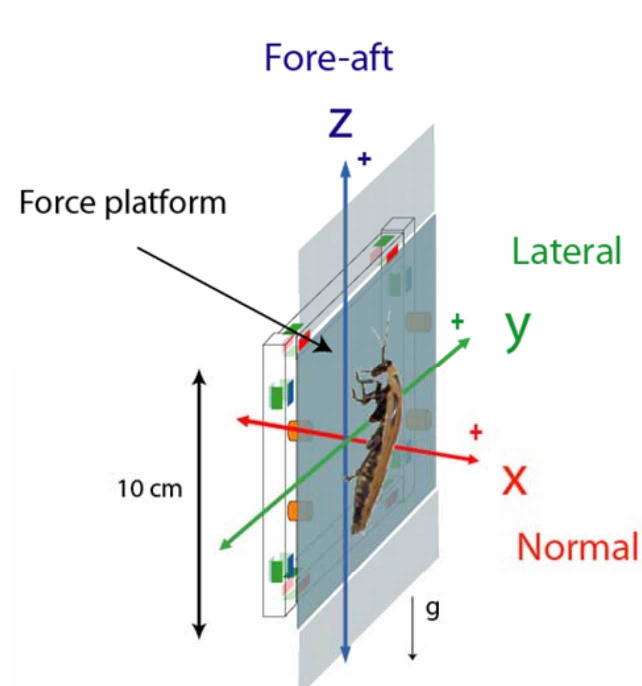




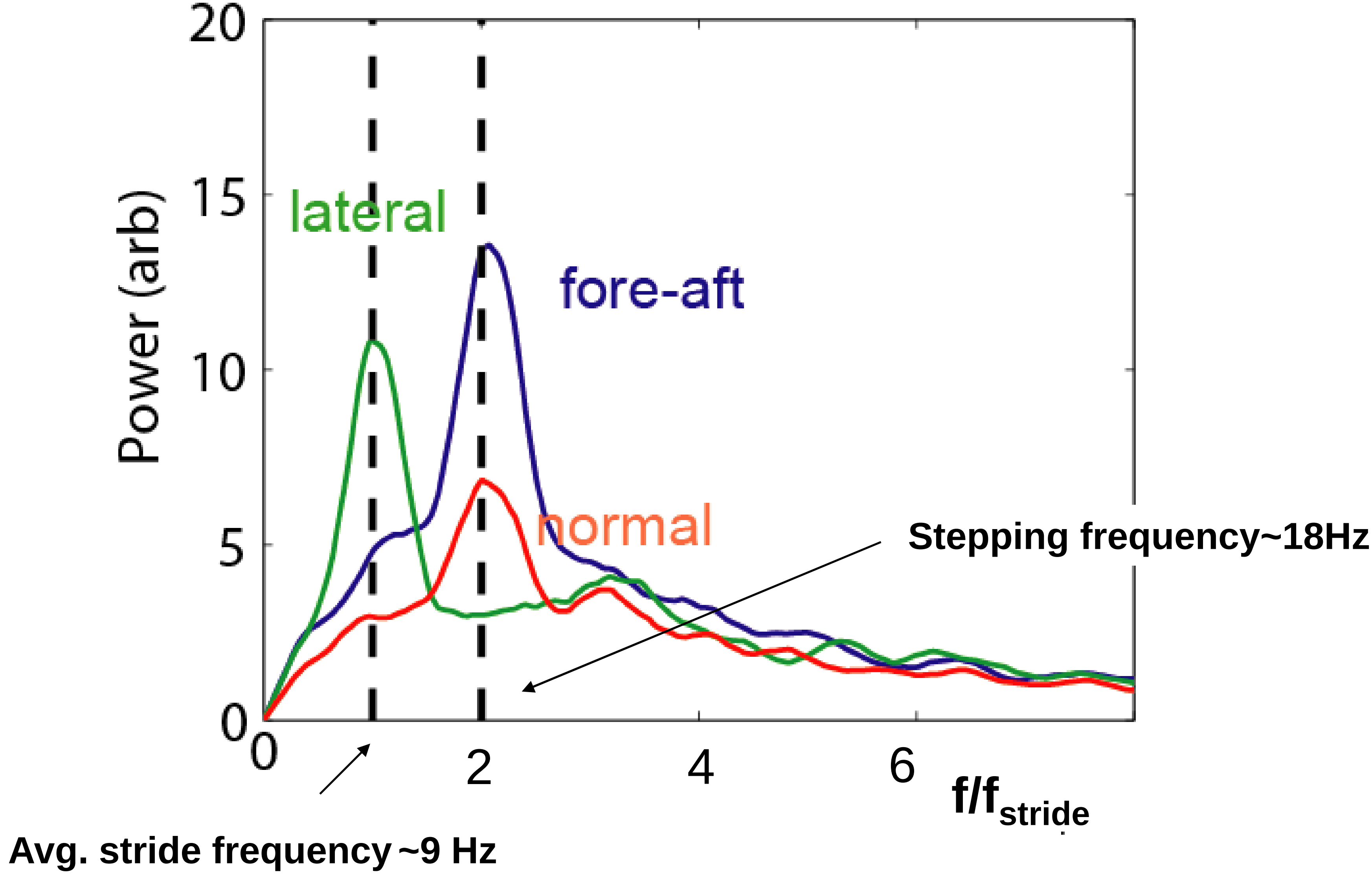
First measurements of forces during rapid climbing of an insect

Goldman, Chen, Dudek, Full
J. Exp. Biol. 2006

CENTER OF MASS (COM) DYNAMICS

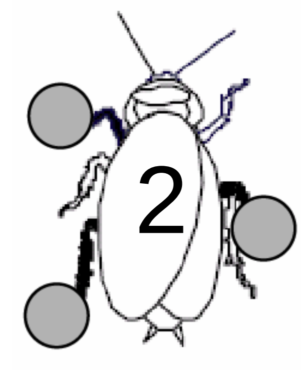


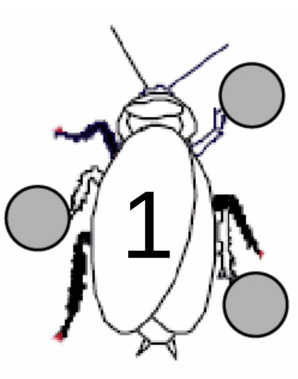
Amplitude in center of mass (COM) force fluctuations

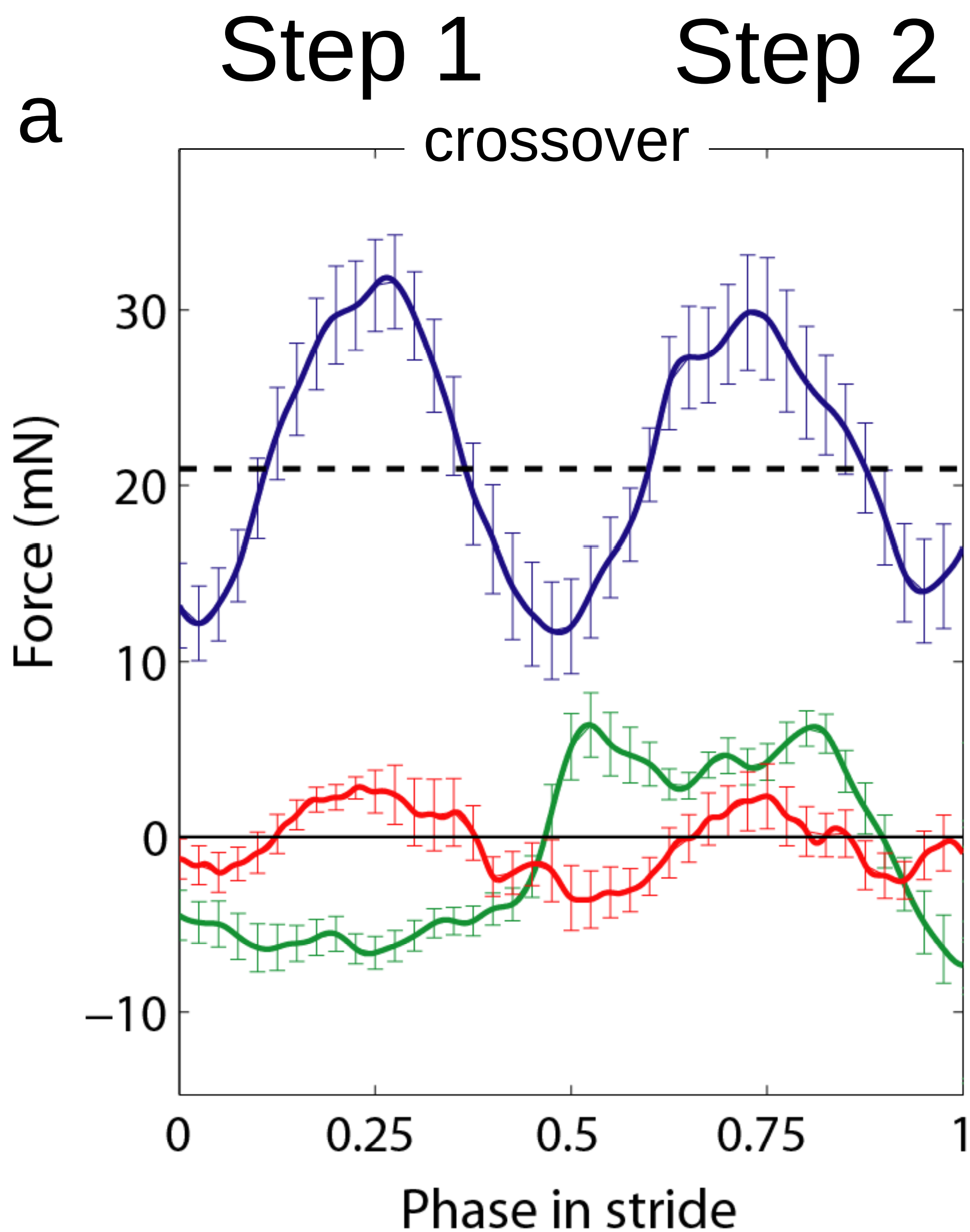


Whole body force data for a **single stride** (two steps)

First measurement of insect rapid climbing forces

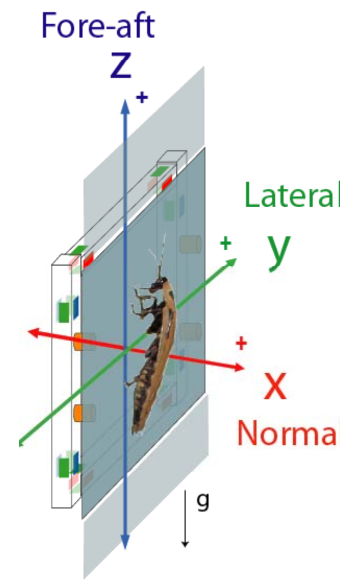






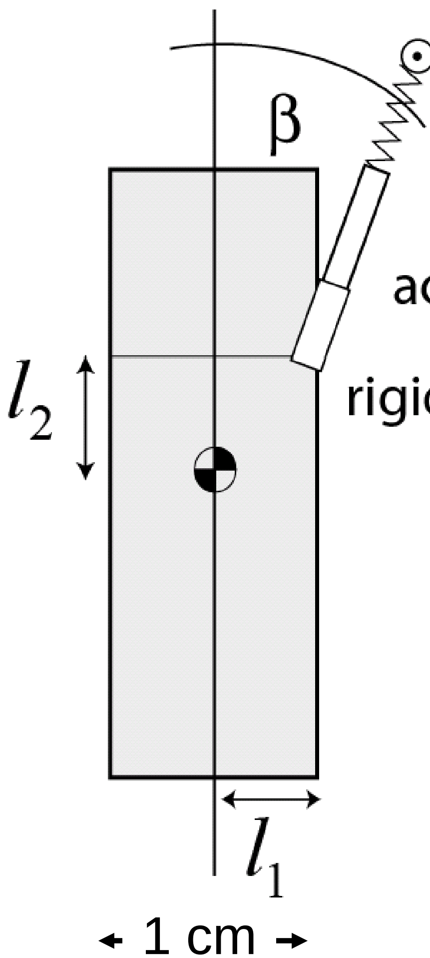
BW

Goldman, Chen, Dudek, Full, *J. Exp. Biol.* 2006



Goldman, Chen, Dudek, Full JEB 2006

Model of climbing cockroach



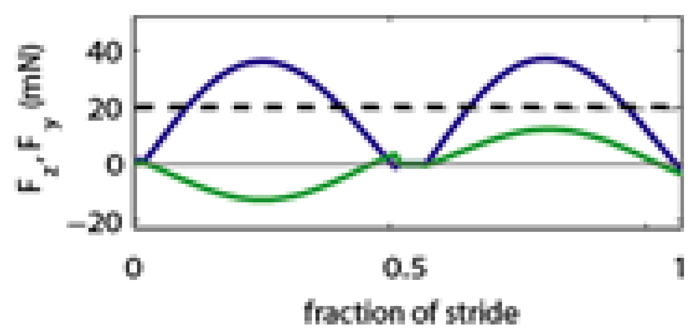
pin joint contact

linear spring

actuator $L(t) = L_0[1 + z\sin(2\pi f t)]$

rigidly attached shoulder

L_0=1.54 cm
l_1=0.71 cm
l_2=0.84 cm
z=0.6
k=6 N/m
f=9 Hz



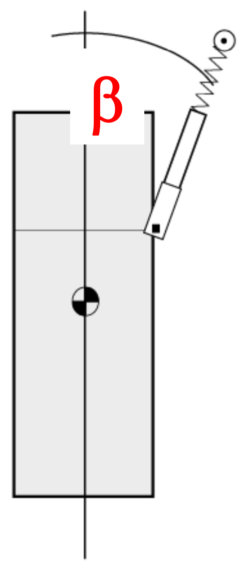
Self-stabilization

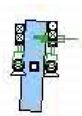
PASSIVE SELF-STABILITY DURING CLIMBING

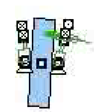
- f_{stride} (lateral osc. freq) ~9 Hz.
- pendulum mode $\sqrt{g/l}$ ~ 3 hz

Small sprawl results in passive stability

β=0 degrees **β=10 degrees**



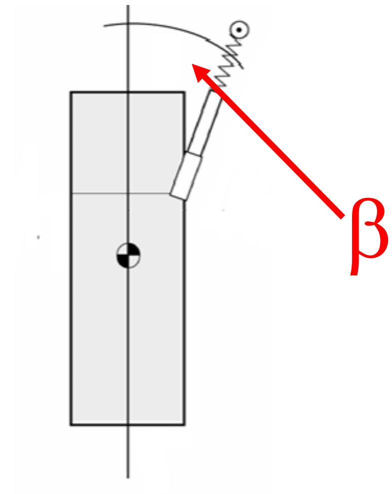


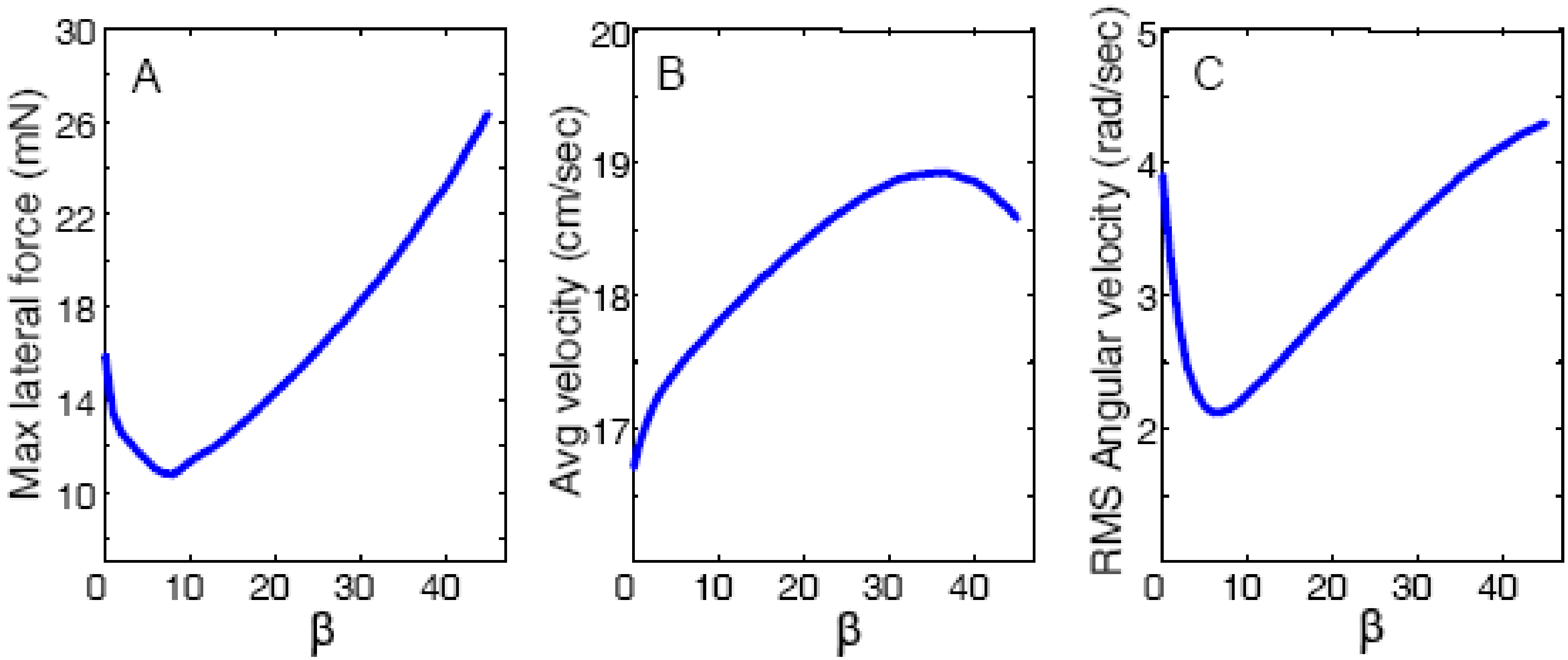


Clark, Goldman, Koditschek & Full 2007

Variation of speed, lateral oscillation and forces with leg angle β

Clark, Goldman, Koditschek & Full 2007

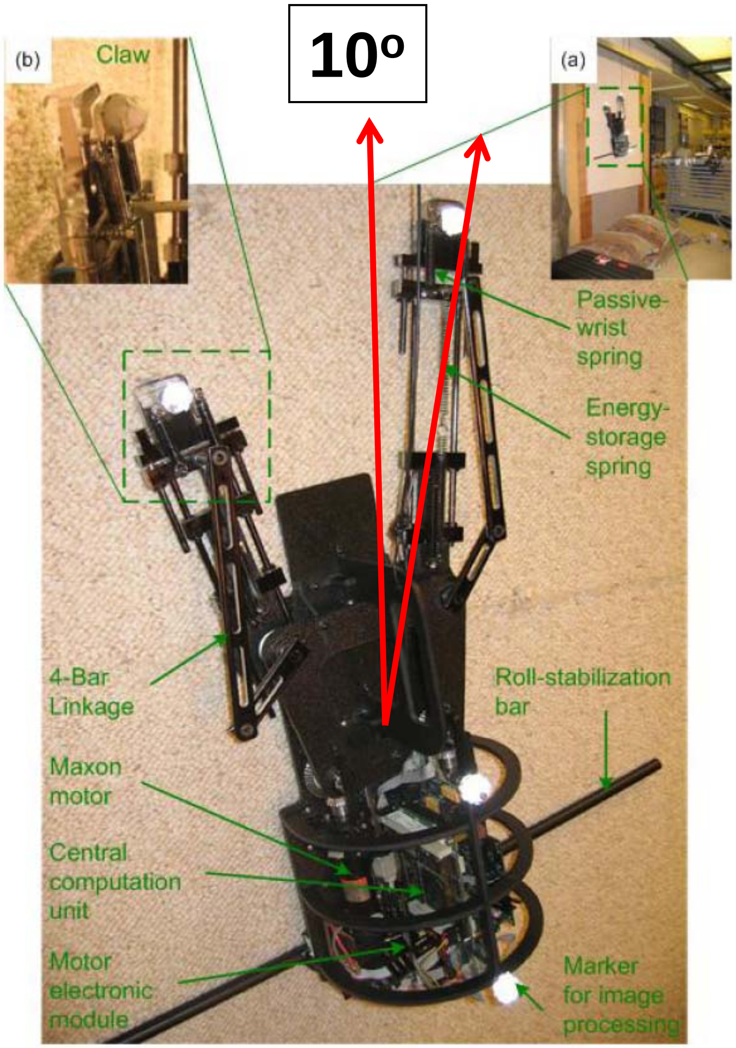




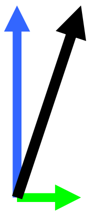
Formal stability analysis needed…

Self stabilization in a physical model

Clark, Goldman, Koditschek & Full 2007



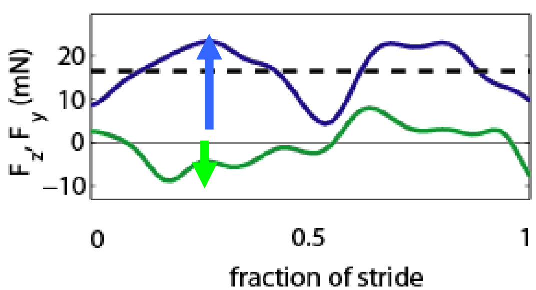
We claim: template for vertical locomotion



~14°

Cockroach

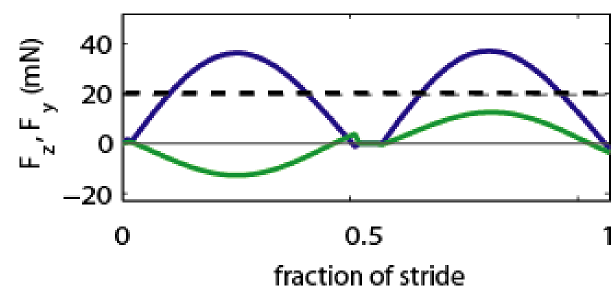
A

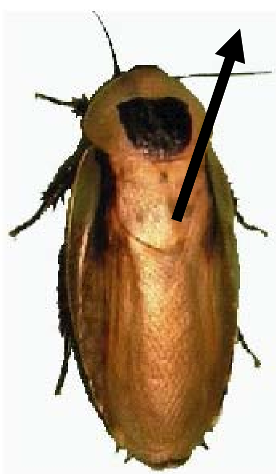


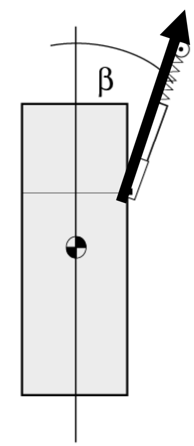
9°

model

C



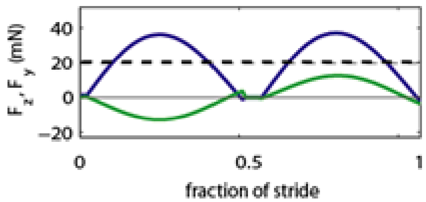


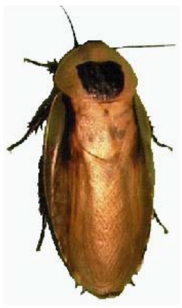


YES!

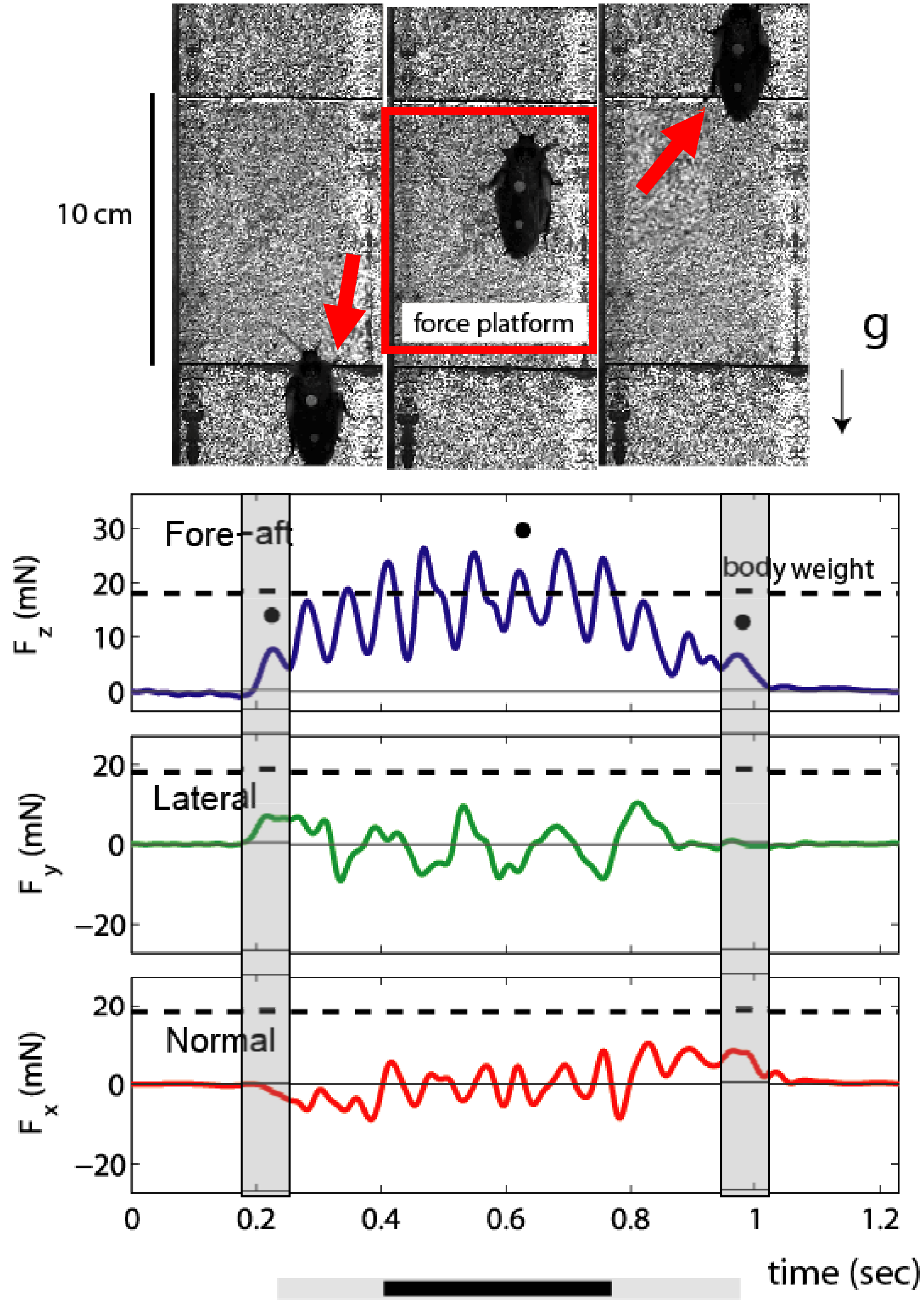
Goldman, Chen, Dudek, Full
J. Exp. Biol. 2006

Template: simple dynamics such that diversity of organisms during a behavior 1) control limb forces to 2) target this dynamics which 3) simplifies control of the behavior





Template: simple dynamics such that diversity of organisms during a behavior 1) control limb forces to 2) target this dynamics which 3) simplifies control of the behavior

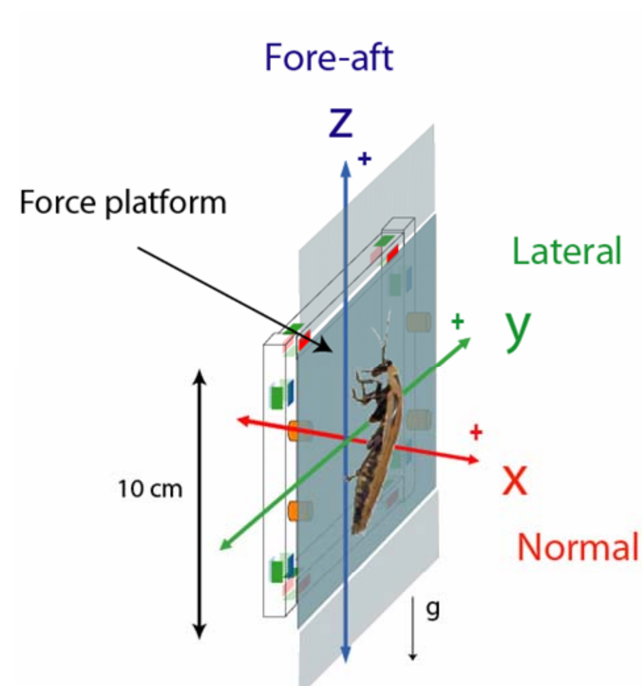


fully on the plate

Measuring single leg forces

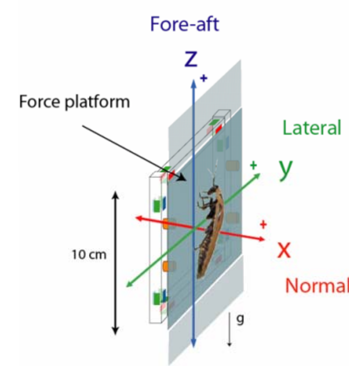
Goldman, Chen, Dudek, Full
J. Exp. Biol. 2006

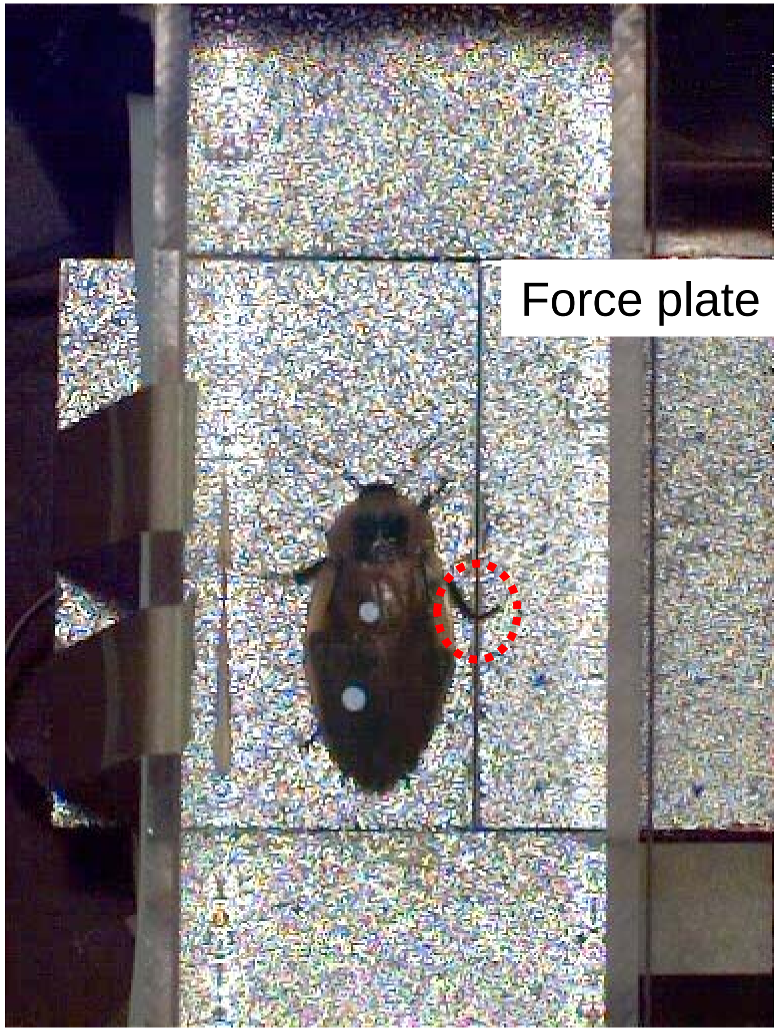
SINGLE LIMB GROUND REACTION FORCES



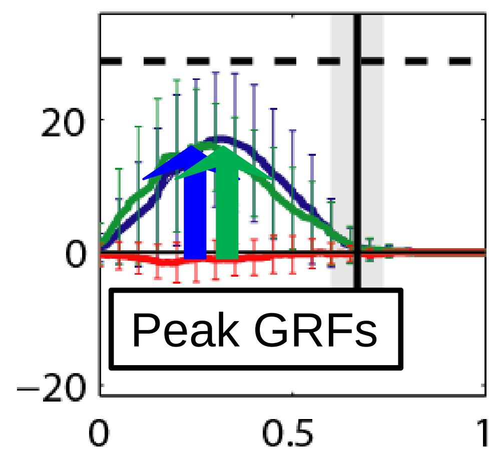
Single Leg Wall Reaction Forces

Goldman, Chen, Dudek & Full, *J. Exp. Biol.* 2006





Middle right limb

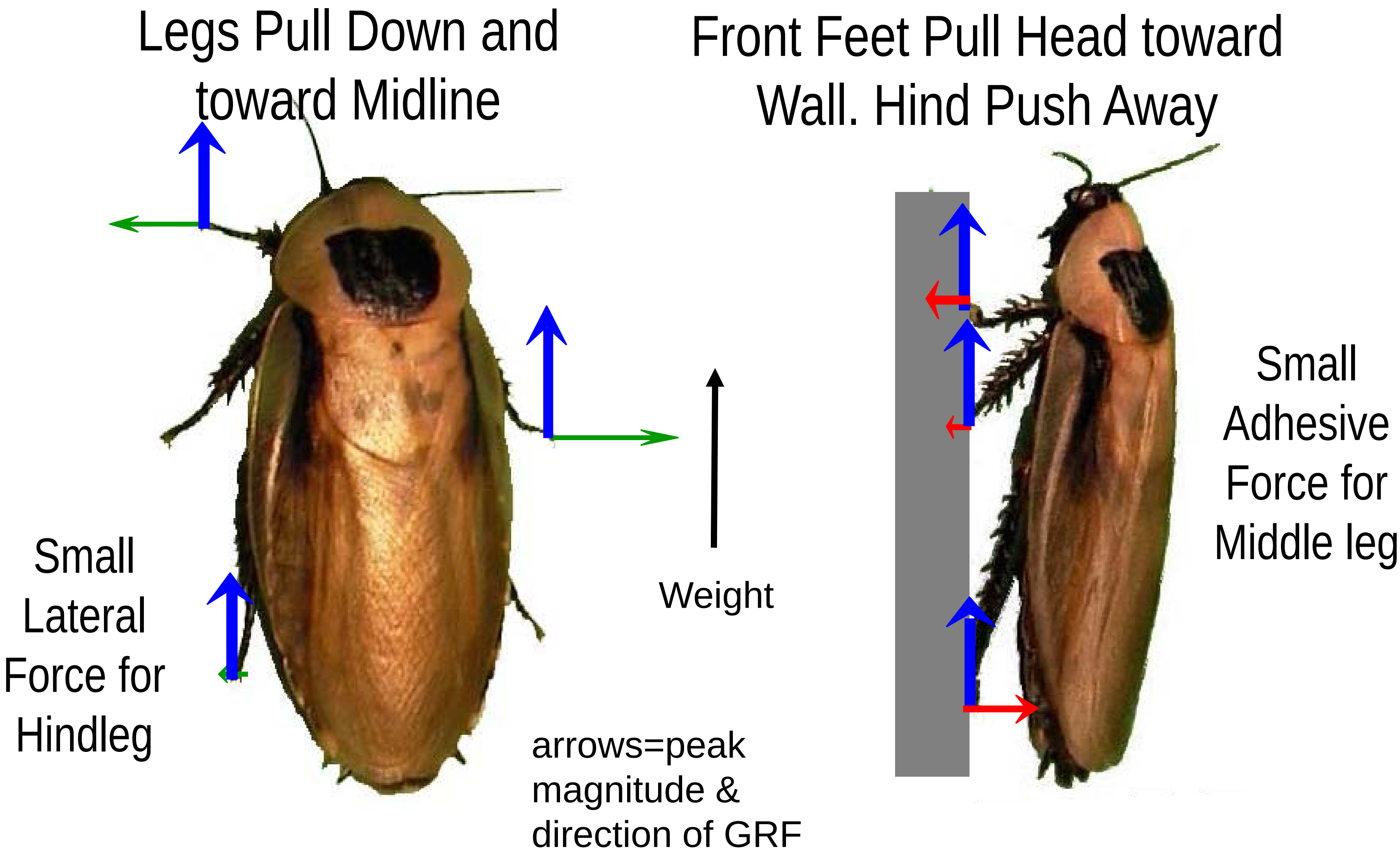


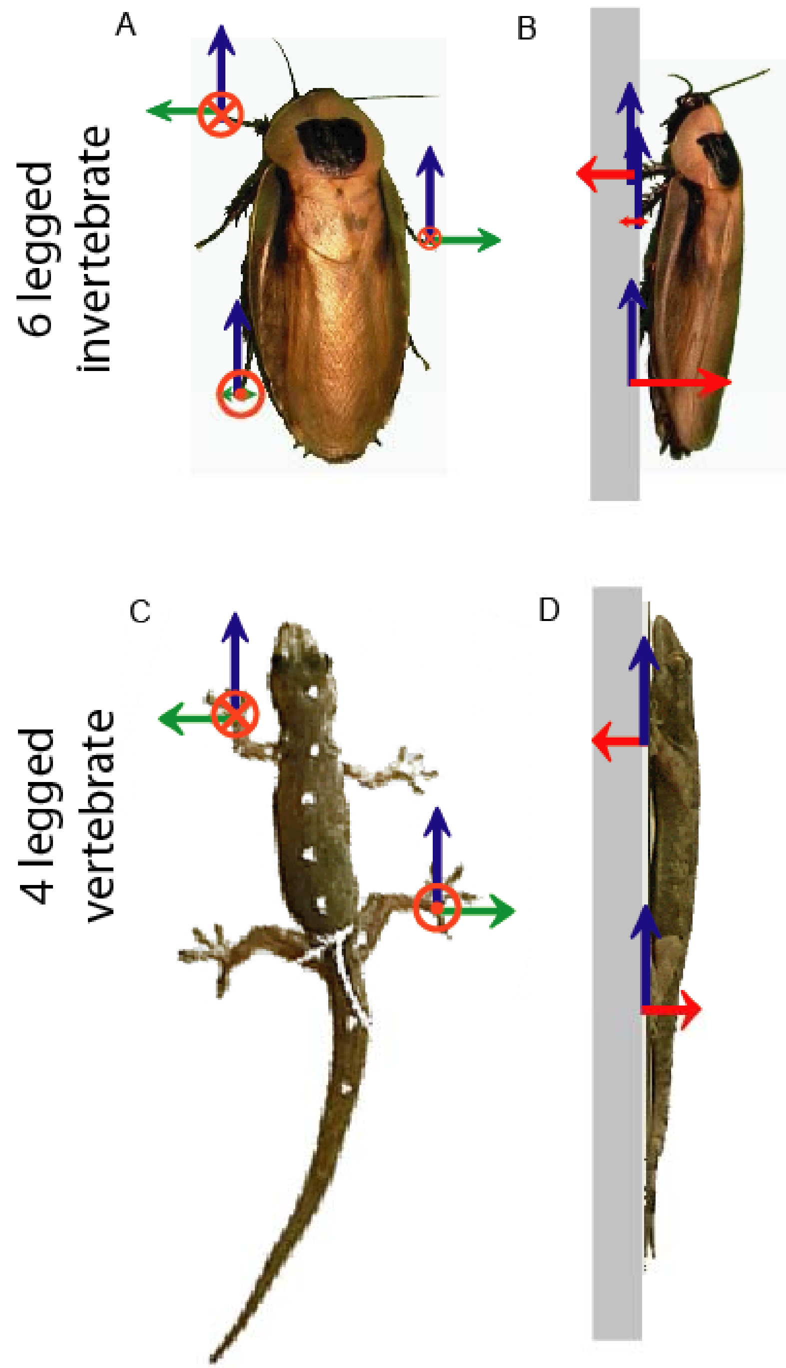
Phase of stride

N=10 animals,
n=52 steps

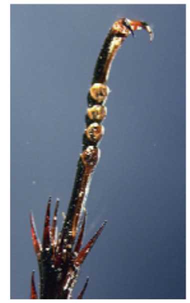
Differential Leg Function

Goldman, Chen, Dudek & Full, *J. Exp. Biol.* 2006





Goldman, Chen, Dudek & Full, *J. Exp. Biol.* 2006

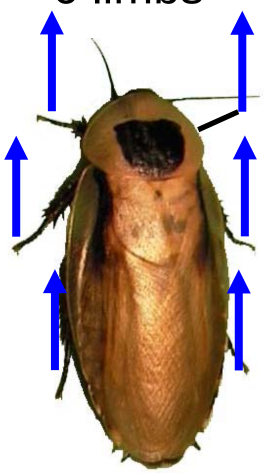


Similar limb use (with different limb number, “toe” number, adhesive mechanism…) to generate similar COM dynamics

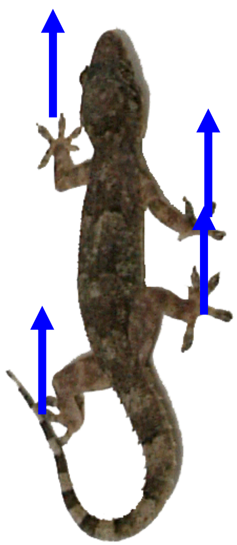
Autumn el at, *J. Exp. Biol.* 2006

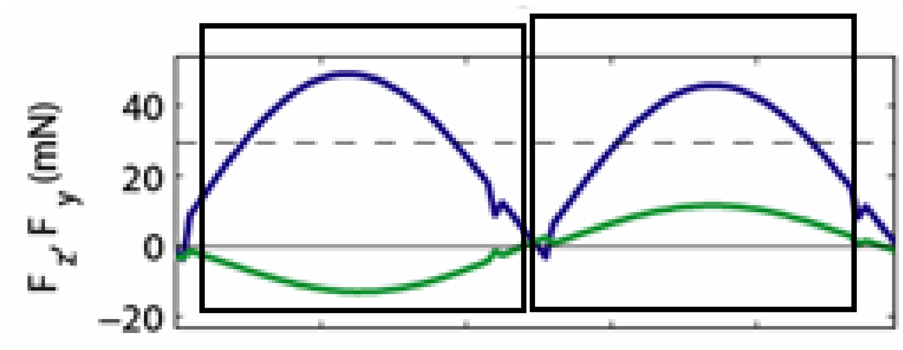
Fore-aft COM dynamics generated by pull from all contacting legs

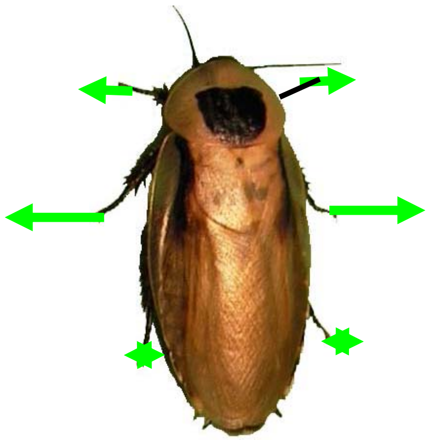
6 limbs

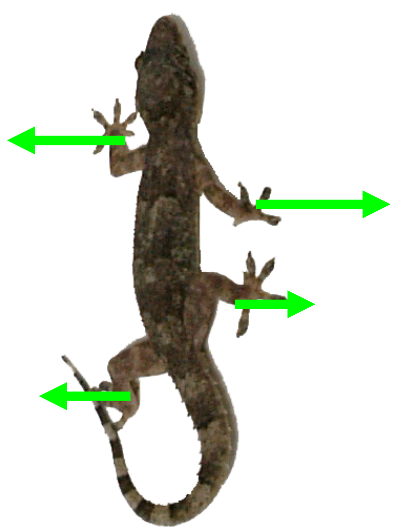


4 limbs

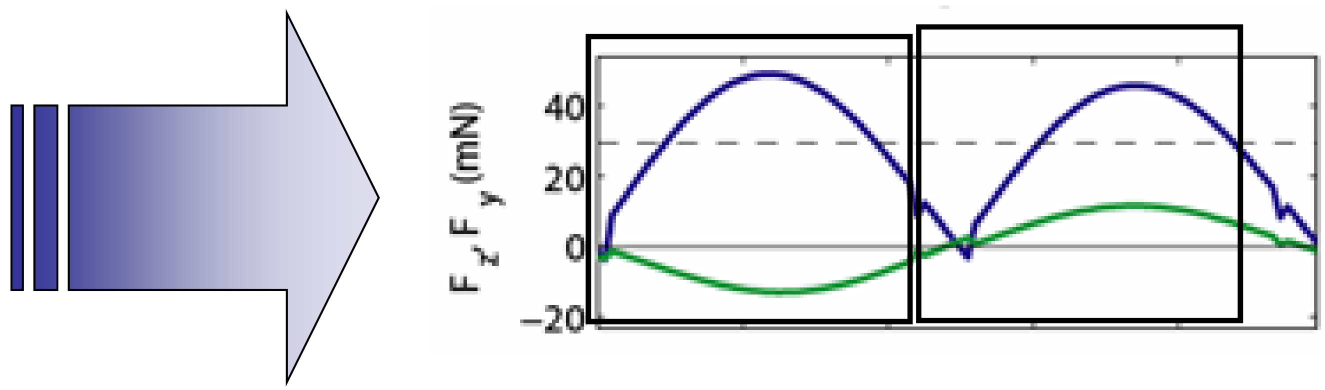








Lateral COM dynamics generated by unbalanced lateral pull-in forces

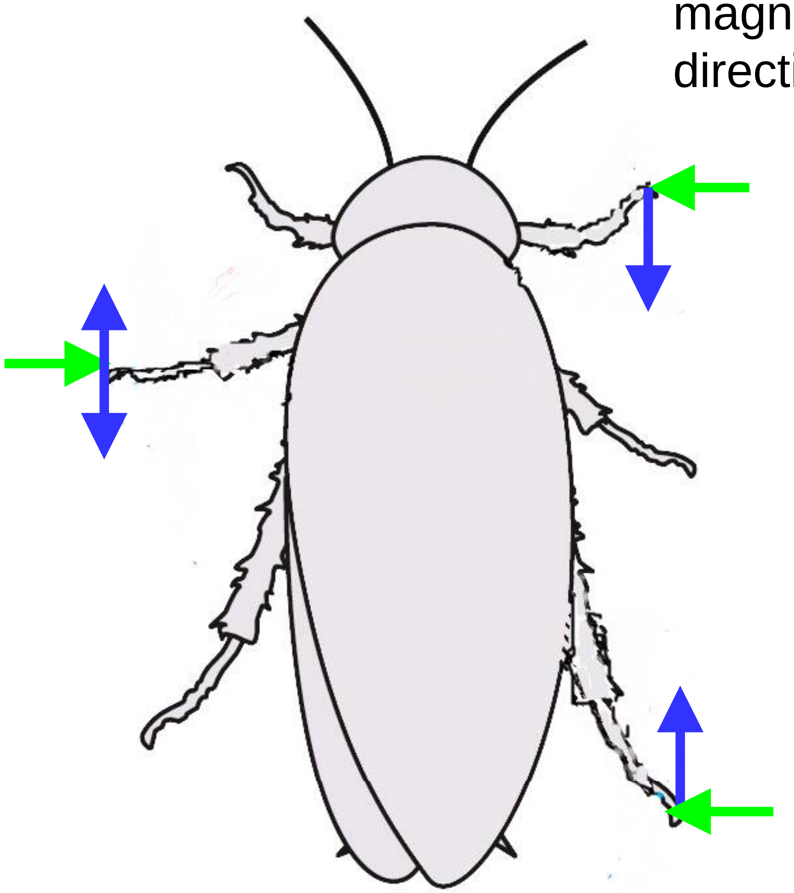


Further evidence for template: limb control

Level Running

Push Out

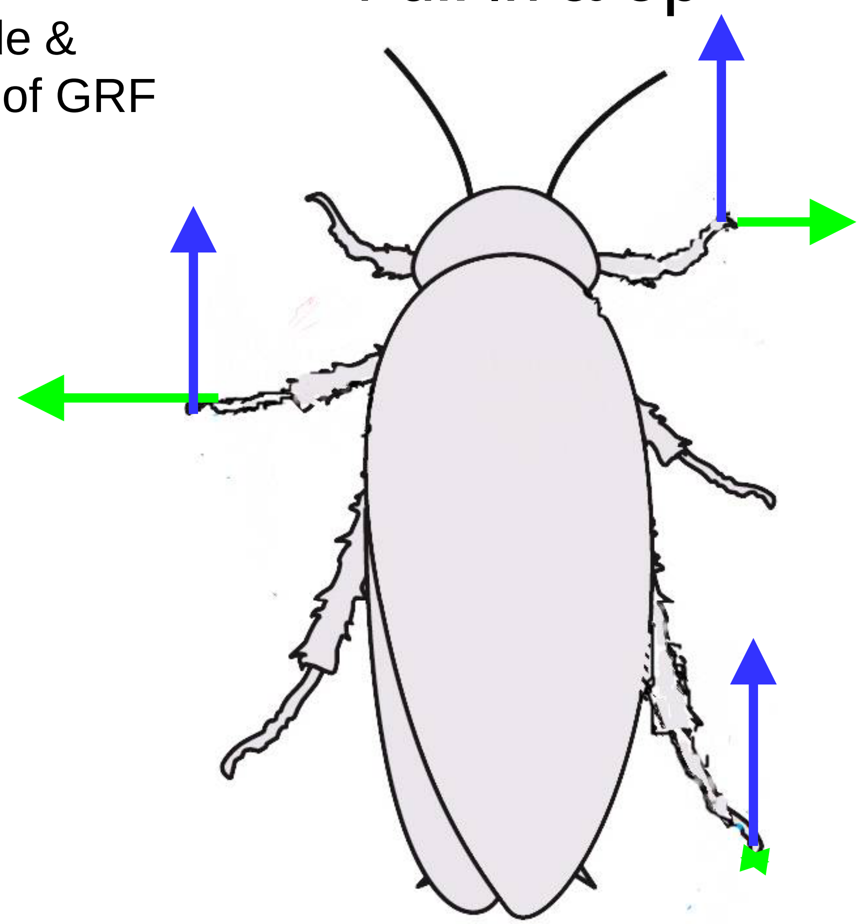
arrows=peak magnitude & direction of GRF



Full, Blickhan & Ting JEB 1991

Vertical Climbing

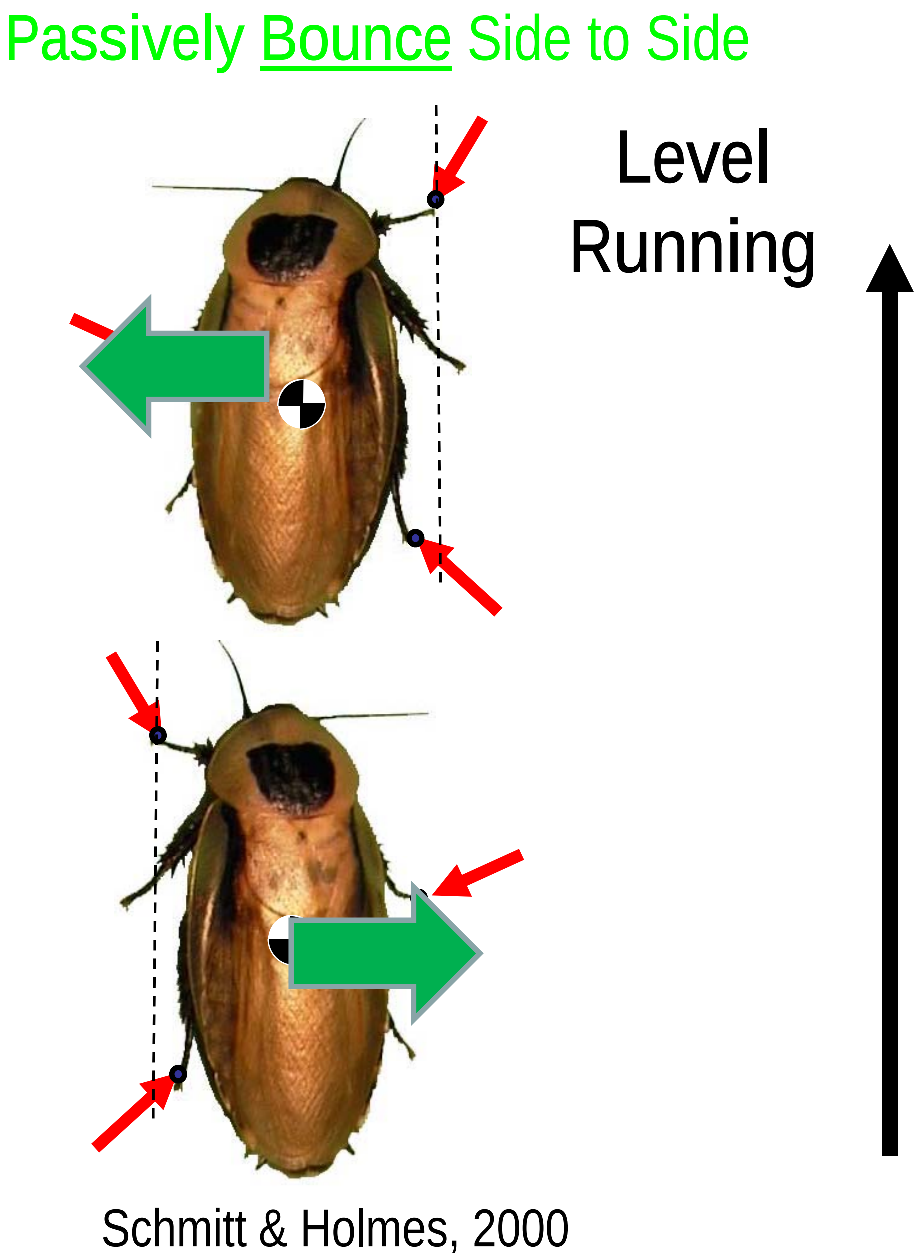
Pull In & Up

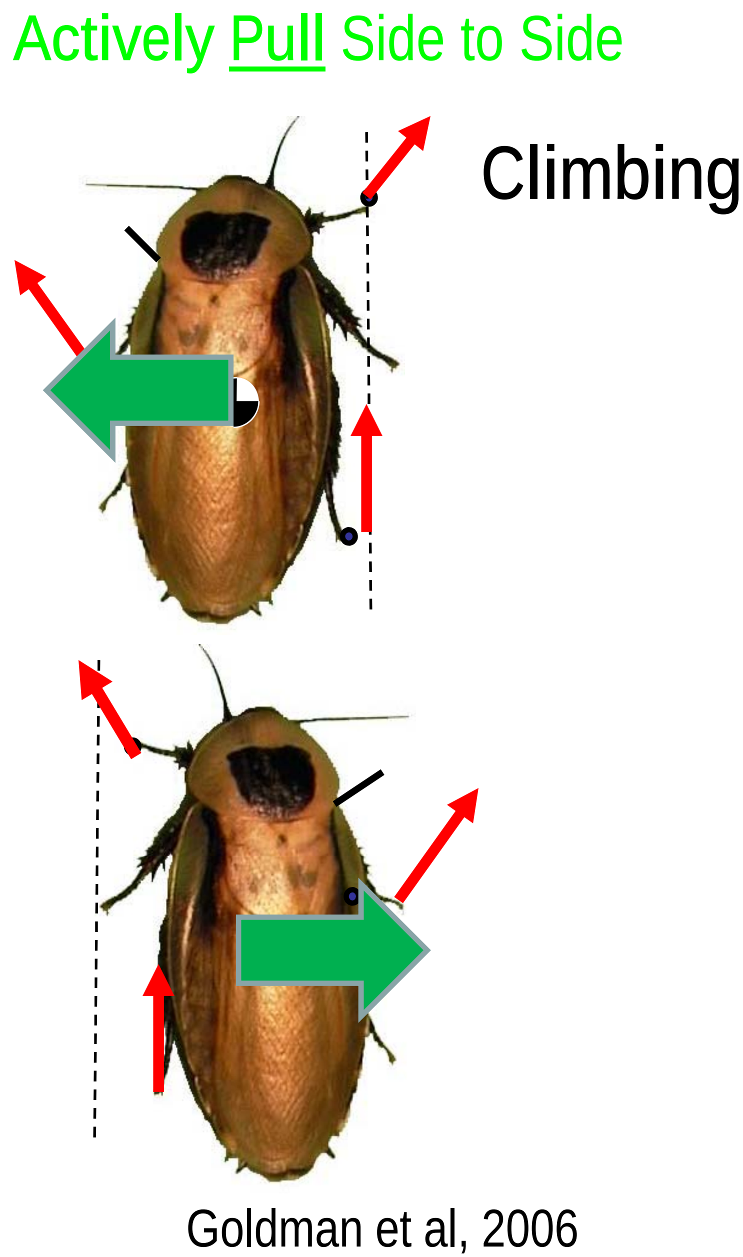


Goldman et al, *JEB.* 2006

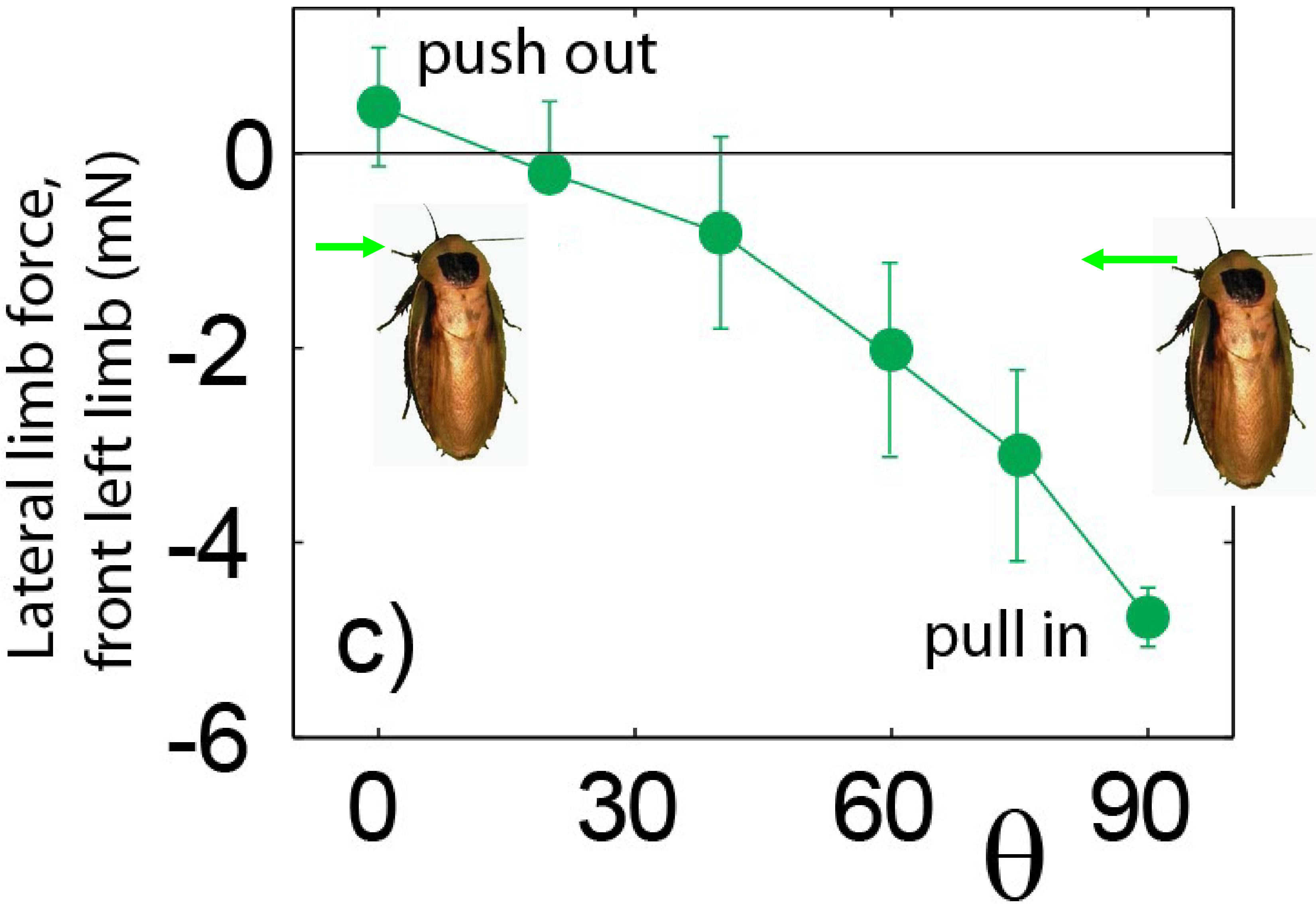
Template: simple dynamics such that diversity of organisms during a behavior 1) control limb forces to 2) target this dynamics which 3) simplifies control of the behavior

Similar lateral oscillation with different limb use



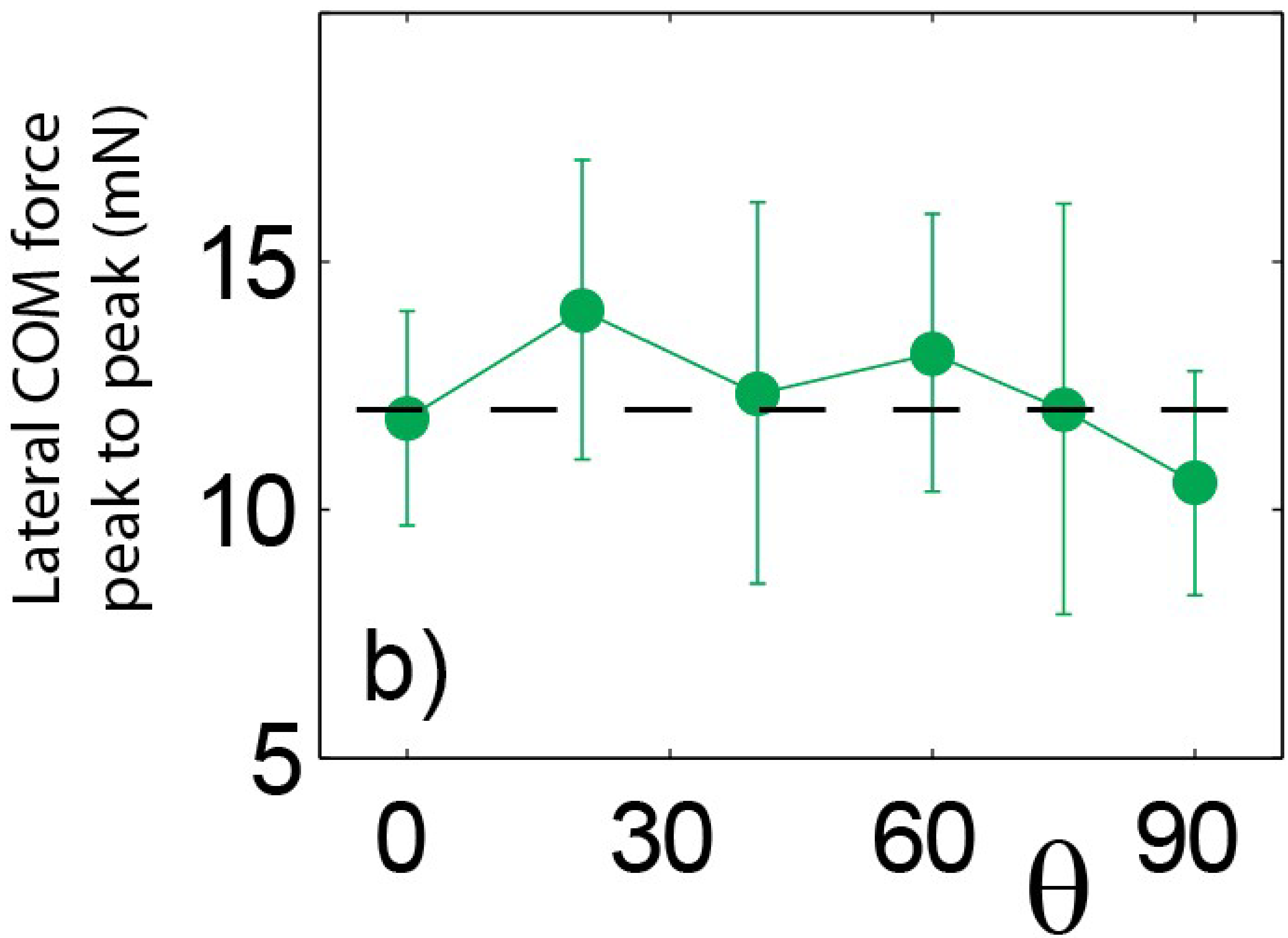


Limb control for lateral oscillation



Template for arbitrary angle?

Side-to-side oscillation conserved, despite changes in limb use



Conclusions

- Control of substrate leads to discovery of principles of locomotion
  - Granular media and climbing
- Discovery of template for rapid climbing with possible self-stabilization properties
  - Similar dynamics observed in 2 species; must test others. Stability analysis needed
- Locomotion studies require techniques and inspire problems at the intersection of biology, physics (nonlinear dynamics), applied math and engineering

Thanks to: Ryan Maladen, Chen Li, Yang Ding, Tao Chen, Robert J. Full, Daniel E. Koditschek, Paul Umbanhowar