The irreversibility and stability of quantum system in terms of classical limit
Tricky for the situation mixed with chaos and regular

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Problems and research tool

One: How to know the likelihood for coming back to initial state in terms of characterizing irreversibility? Two: Quantum motion is non-chaotic but how to obtain the fingerprint of classical chaotic motion and regular one in quantum mechanics?

Recently the community of quantum chaos is developing a new idea to treat such hard problems and the key point is to disturb the Hamiltonian rather than to disturb the initial state just done in classical chaos. The internal product of quantum states can fulfill the job, this kind of definition gives a very powerful research tool so called Loschmidt echo.

$$M(t) = |m(t)|^2 = \left|\langle \Psi_0 | \exp(iHt/\hbar)\exp(-iH_0 t/\hbar) | \Psi_0 \rangle\right|^2$$

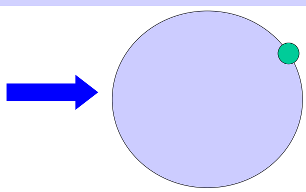
$$H = H_0 + \varepsilon V$$

The mixed type phase space in terms of classical and quantum corresponedence attracts us very much, in particular for two cases with very few research works previously done: the edge of chaos and transition from weak chaos to strong chaos.

Research model

Kick rotator:

Consider a particle moving along a circle. The friction and gravity are negligible. Kicks of period T=1 are applied to the particle.



Classical world
Hamiltonian canonical equation

Quantum world
Floquet evolutive operator

$$H = \frac{1}{2}p^2 + V(r)\sum_n \delta(t - nT)$$

$$\begin{cases} r_{n+1} = r_n + p_n (\mathrm{mod}\, 2\pi) \\ p_{n+1} = p_n + K \sin r_{n+1} \end{cases}$$

$$\hbar_{eff} = 2\pi/N$$

N Dimension of Hilbert space

$$U = \exp[-i\,\hat{p}^2/(2\hbar)]\exp[-iV(\hat{r})/\hbar]$$

- **Typical dynamic properties in terms of mixed phase space**

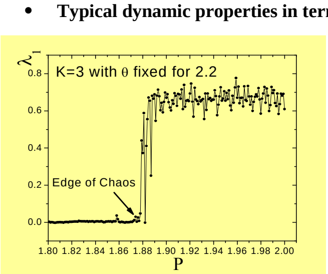


Fig. 1: The Largest Lyapunov exponent λ_1 versus momentum p for the fixed nonlinear parameter $K = 3$ as well as angle $r = 2.2$. The time for computing is basically order of 10^3.

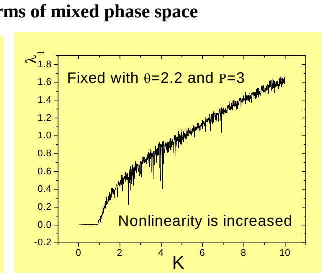


Fig. 2: The Largest Lyapunov exponent λ_1 versus nonlinear parameter K with the fixed angle and momentum $p = 3$, $r = 2.2$. The time step for computing is basically order of 10^3.

Research methods

- **Numerical simulation**

Fast Fourier Transform is employed (variation of quantum presentations)

Eigenstates of $\hat{r}$ are denoted by $|j\rangle$
$\hat{r}|j\rangle = j\hbar|j\rangle$, $\psi(t) = U^t \psi_0$
Same formulation for Eigenstates of $\hat{p}$
Perturbation $\sigma = \varepsilon/\hbar$, $\varepsilon << K_0$

- **Theoretical analysis**

Semi-classical path integral ($\hbar \to 0$)

$$\overline{M}(t) \equiv \overline{|m(t)|^2} = \overline{M}_a(t) + \overline{M}_f(t),\ \overline{M}_a(t) \equiv |\overline{m}(t)|^2,\ \overline{m}(t) = \frac{1}{v}\int d\vec{r}_0\, m(\vec{r}_0, t)$$

$$\overline{M}_a(t) \approx \left|\int d\Delta S e^{i\Delta S/\hbar} P(\Delta S)\right|^2,\ \Delta S(\vec{p}_0, \vec{r}_0; t) = \varepsilon\int_0^t dt' V[\vec{r}(t')],\ M_{SC}(t) = \exp\left[-2(\varepsilon/\hbar)^\eta D_L\right] \text{(Levy distribution)}$$

Levy distribution: $L(x,\eta,\beta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_L(z) e^{izx} dz$, $F_L(z) = \exp\left\{-igz - D_L|z|^\eta\left[1 + i\beta\,\mathrm{sgn}(z)\omega(z,\eta)\right]\right\}$

Typical Results

- **Decay of Loschmidt echo for typical field of edge of chaos**

We focus on the specific case of edge in terms of Fig.1 and use the wave packet as our initial quantum state. To give a relatively clear feature, we select several different initial wave packets corresponding to edge of chaos($K = 3$, $r = 2.2$, p has the variation from basically 1.87 to 1.89). We use reference lines to show the decay laws fitting the approximate slope-which corresponds to decay exponent.

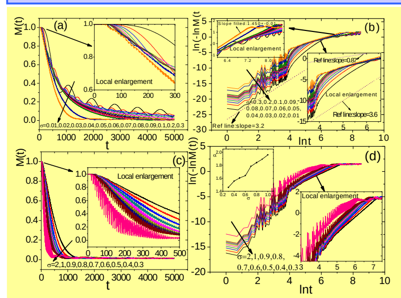


Fig.3: The initial position of wave packet $r = 2.2$ and $p = 1.876$. (a) and (c) shows the decay behaviors for typical perturbations, (b) and (d) shows decay law using the relationship $\ln(-\ln M(t))$ versus $\ln t$, $\sigma = 0.3$ corresponds to the disappearance of the intermediate process. The left inset of (d) shows the approximate decay exponents using numerical fitting.

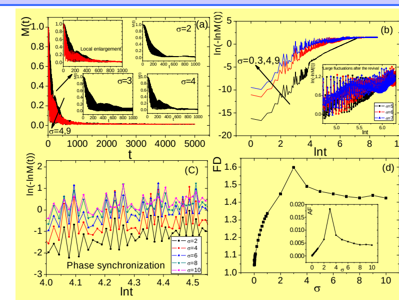


Fig.4: The same initial position of wave packet to the Fig3.(a) shows the typical decay in terms of large perturbation, the inset shows the sharp enlargement of fluctuation corresponding to the adjacent region of $\sigma = 3$.(b) shows the decay laws and the inset illustrates the big fluctuation after the revival. (c) shows the phase synchronization and (d) shows the variation of Fractal dimension (FD) and average Fluctuation (AF) with the enlargement of perturbation.

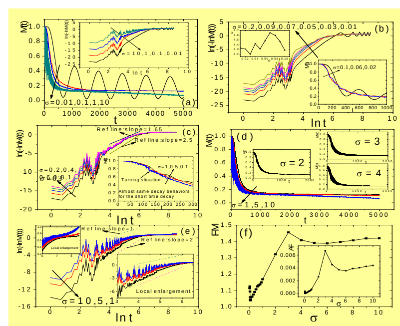


Fig. 5: The initial position of wave packet $r = 2.2$ and $p = 1.882$. (a) shows the typical decay processes with the inset giving the decay laws generally. (b) shows the decay laws for small perturbation, the left inset shows the decay exponents. (c) shows the decay law for intermediate perturbation. (d) and (e) shows the typical decay processes in terms of large perturbation and related decay laws. (f) shows the variation of FD and AF versus different perturbation.

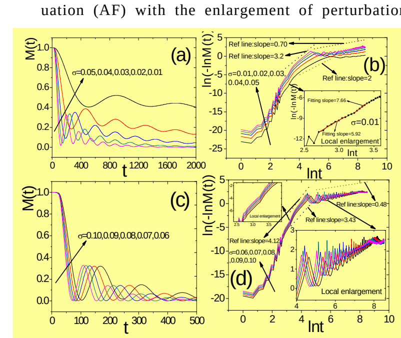


Fig. 6: The initial position of wave packet $r = 2.2$ and $p = 1.886$. (a) and (c) shows the typical decay processes for small perturbation distinguished from emergent situation for revival after saturation. (b) shows the decay laws for (a) and the inset illustrates the specific intermediate process in terms of $\sigma = 0.01$. (d) shows the decay laws for (c), and the folding situation can be found in the left inset.

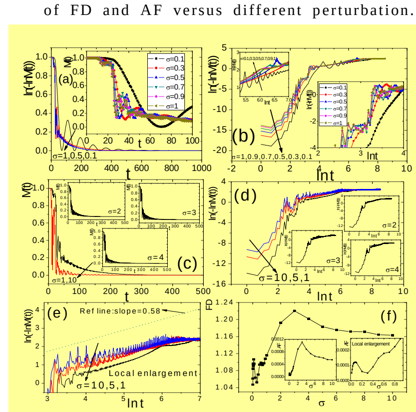
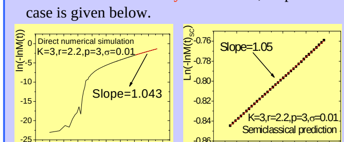


Fig. 7: The same initial position of wave packet to Fig.6.(a) shows the decay processes for intermediate perturbations and the inset shows the decay behaviors of short time scale with a sharp drop followed by a gradually common decay process in terms of perturbation beyond $\sigma = 0.3$.(b) shows the decay laws corresponding to intermediate perturbation, the turning situation can be found very clearly in the right inset and the left inset shows the divergent situation after basically common decay process. (c) shows the decay behaviors for large perturbations and (d) shows the related decay laws with (e) to give the illustration for the slow decay process clearly. (f) shows the variation of FD and AF versus perturbation.

- **Conclusion and further research**

The basic picture for the decay rules in terms of edge of chaos are the follows: Firstly, the initial decay process shows the big decay exponent(numerical fitting meaning) which has no strong function for small perturbation (can not observed the decay obviously from naked eyes) but is taking account for the initial sharp decay process corresponding relatively large perturbations afterwards. Secondly, for small perturbations, there has a transient decay process showing good decay law(straight line for $\ln(-\ln M(t))$ versus $\ln t$) or not, and this kind of decay section can be shortened by increasing perturbation. Thirdly, for intermediate perturbation, we can expect the good decay laws in most cases. Fourthly, for large perturbations, the fluctuations undergo a burst change around $\sigma = 3$ which can be found to have the basic pattern using FD and AF.

Now we want to explain these numerical works, and find it has a long way to go. Regarding to the transition from weak chaos (wave packet in the chaotic sea) to strong chaos, we begin to get some good theoretical prediction coming from the consideration of statistical properties of classical action for Levy distribution, a specific case is given below.



Reference

Qiang Zheng, Wen-ge Wang, Pinquan Qin, et al, Decay of Loschmidt echo in a Bose-Einstein condensate at a dynamical phase transition,Phys.Rev.E.80,016214 (2009)

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